Prep [3] - Second Term - Algebra - Unit [1] - Equations

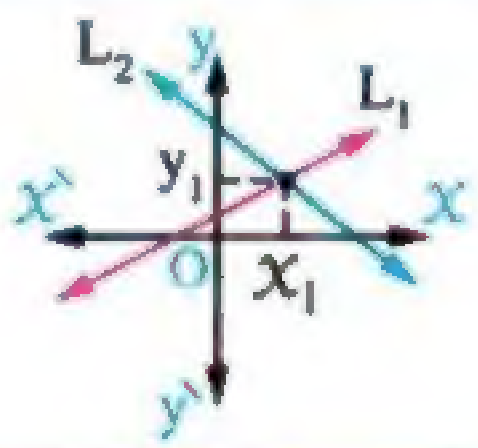
Lesson [1] : Solving Two Equations Of First Degree In Two Variables

First : Graphically

Then to solve the two equations graphically, we do as follows :

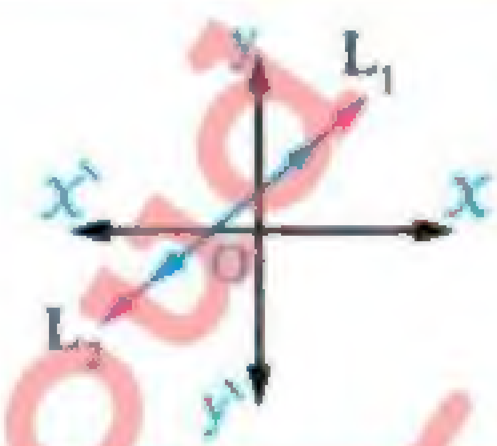
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 **intersect** at the point (x_1 , y_1)



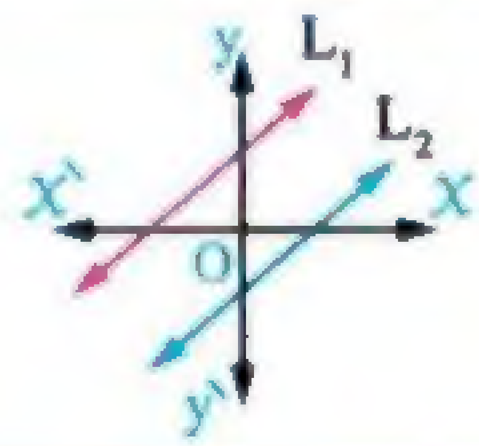
- There is a unique solution (x_1 , y_1)
- The S.S. = $\{(x_1 , y_1)\}$

2 L_1 and L_2 are **coincident**



- There is an infinite number of solutions

3 L_1 and L_2 are **parallel**



- There is no solution
- The S.S. = $\varnothing$

Remark : -

Determining the number of solutions without graphing

First : Find the slopes of the two straight lines m1 and m2

<table>
<tr><th>m1 ≠ m2</th><th colspan="2">m1 = m2</th></tr>
<tr><td rowspan="2">Then the two straight lines intersect at one point, and then the number of solutions = 1</td><td colspan="2">Then find the points of intersection of the two straight lines with y-axis</td></tr>
<tr><td>The two points are equal
Then the two straight lines are coincident , and then the number of solutions is an infinite number.</td><td>The two points are different
Then the two straight lines are parallel , and then the number of solutions = 0</td></tr>
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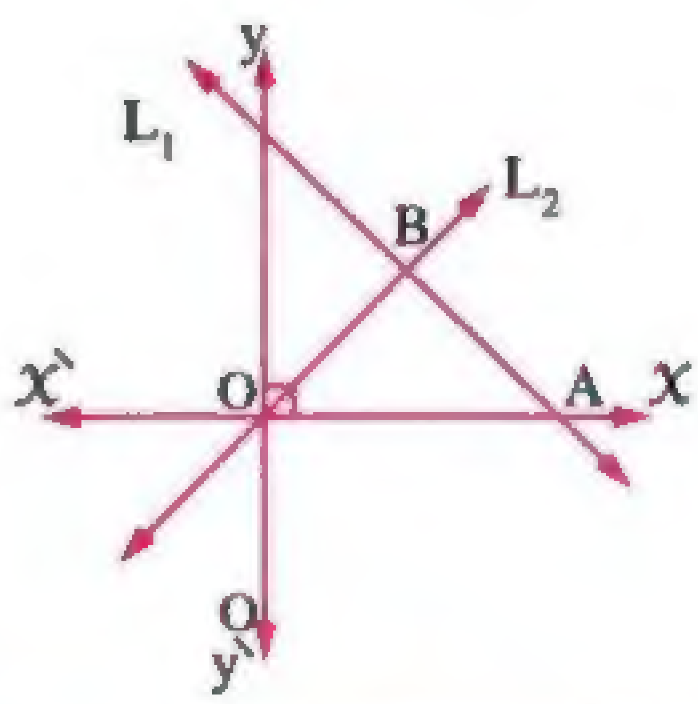
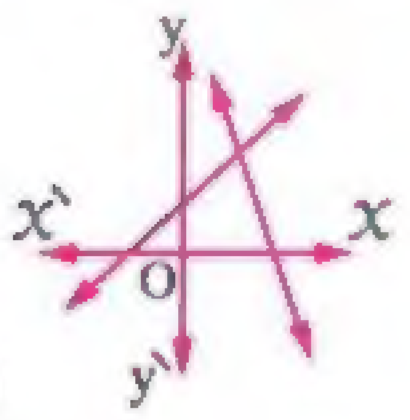
Exercises

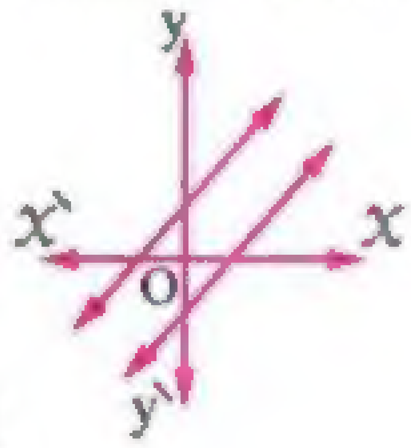
[A] Essay problems : -

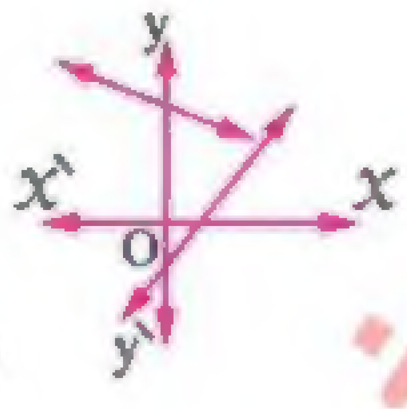
1	Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations : $y + x = 7$, $y = 2x + 1$ (Alexandria 15) « {(2 , 5)} »
2	Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations : $x + y = 5$, $x - y = 1$ (South Sinai 13) « {(3 , 2)} »
3	Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations : 📖 $3x + y = 5$, $y + 3x = 8$ « $\varnothing$ »
4	Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations : 📖 $2x + y = 4$, $8 - 2y = 4x$ « an infinite number »
5	Find graphically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations : 📖 $2x + y = 0$, $x + 2y = 3$ « {(– 1 , 2)} »
6	What is the number of solutions of each pair of the following equations : 📖 $7x + 4y = 6$, $5x - 2y = 14$
7	What is the number of solutions of each pair of the following equations : 📖 $9x + 6y = 24$, $3x + 2y = 8$
8	**In the opposite figure :** If the equation of straight line $L_1 : x + y = 6$ and the equation of the straight line $L_2 : y - 2x = 0$ where $L_1 \cap L_2 = \{B\}$, O is the origin point , $A \in \overleftrightarrow{x x'}$ **Find :** The surface area of the triangle OAB  (El-Sharkia 15) « 12 square units »

[B] Choose the correct : -

1 Which of the following graphs represents two equations of the first degree in two variables which have no common solution ? *(Port Said 19)*

(a) 

(b) 

(c) 

(d) 

2 The point of intersection of the two straight lines : $x + 2 = 0$, $y = x$ is .... *(El-Dakahlia 17)*

(a) (2 , 2) (b) (2 , 0) (c) (– 2 , – 2) (d) (0 , 0)

3 The two straight lines : $3x = 7$, $2y = 9$ are .... *(Matrouh 16 , Luxor 16)*

(a) parallel. (b) coincident.
(c) intersecting and non perpendicular. (d) perpendicular.

4 The two straight lines representing the two equations : $x + 5y = 1$, $x + 5y - 8 = 0$ are .... *(El-Beheira 17 , Giza 16)*

(a) parallel. (b) coincident.
(c) perpendicular. (d) intersecting and not perpendlicular.

5 🕮 The S.S. of the two equations : $x - 2y = 1$, $3x + y = 10$ in $\mathbb{R} \times \mathbb{R}$ is .... *(Souhag 18 , Port Said 13 , El-Fayoum 11)*

(a) $\{(5, 2)\}$ (b) $\{(2, 4)\}$ (c) $\{(1, 3)\}$ (d) $\{(3, 1)\}$

6 🕮 The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersecting at .... *(Alexandria 14 , El-Beheira 11)*

(a) the origin point. (b) the first quadrant.
(c) the second quadrant. (d) the fourth quadrant.

7	If the point of intersection of two straight lines : $x - 1 = 0$, $y = 2k$ lies on the fourth quadrant , then k may be equal .... *(Kafr El-Sheikh 16)* (a) -5 (b) 0 (c) 1 (d) 5
8	The number of solutions of the two equations : $x - \frac{1}{2} y = 4$, $2x - y = 2$ in $\mathbb{R}^2$ is .... *(El-Kalyoubia 16 , El-Monofia 16)* (a) a unique solution. (b) two solutions. (c) an infinite number of solutions. (d) zero.
9	If there are infinite number of solutions in $\mathbb{R} \times \mathbb{R}$ of the two equations : $x + 4y = 7$, $3x + ky = 21$, then k = ... *(Souhag 19 , El-Beheira 18 , Qena 17 , Alexandria 16)* (a) 4 (b) 7 (c) 12 (d) 21

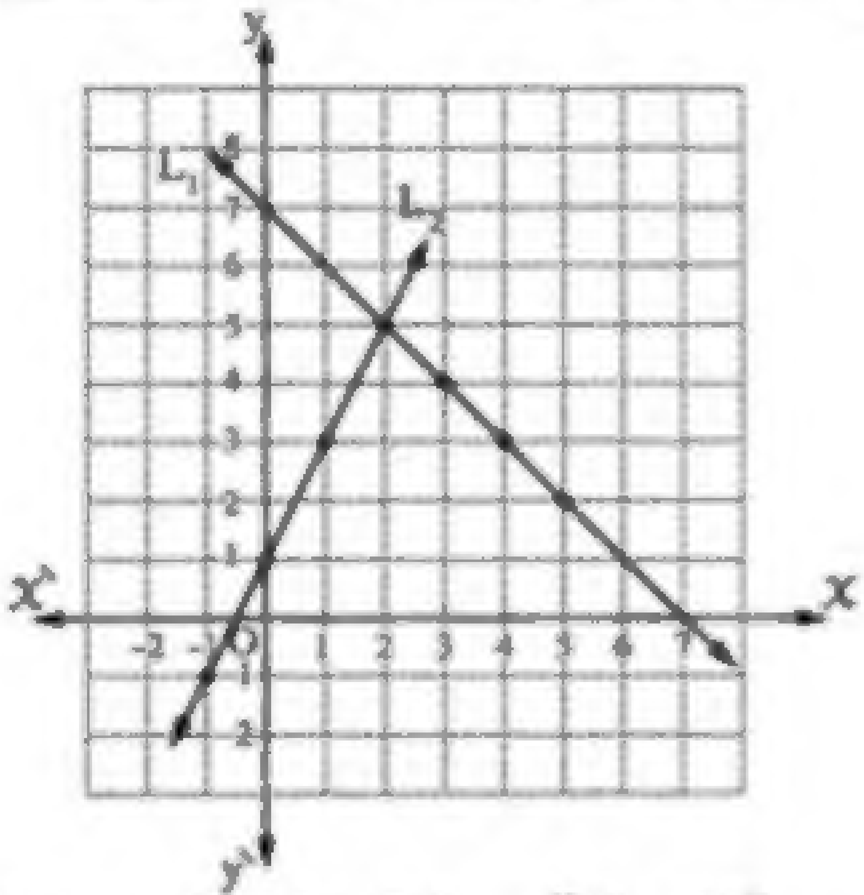
Solutions

1

$y = 7 - x$, $y = 2x + 1$

x	3	4	5
y	4	3	2

x	−1	0	1
y	−1	1	3



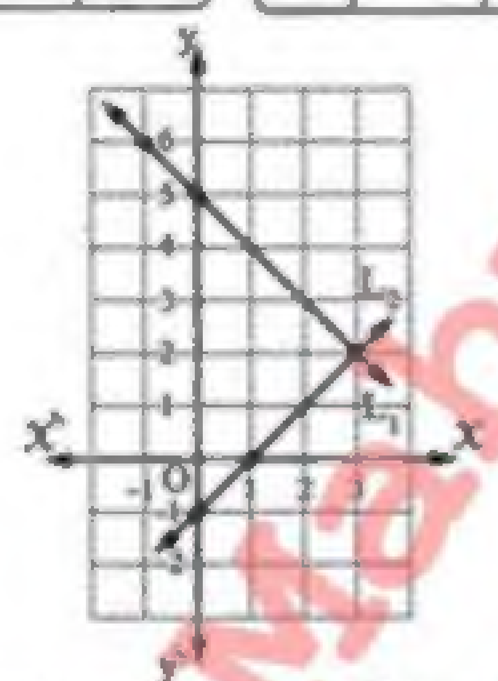
from the graph , the S.S. = $\{(2 , 5)\}$

2

$y = 5 - x$, $y = x - 1$

x	0	−1	3
y	5	6	2

x	0	1	3
y	−1	0	2



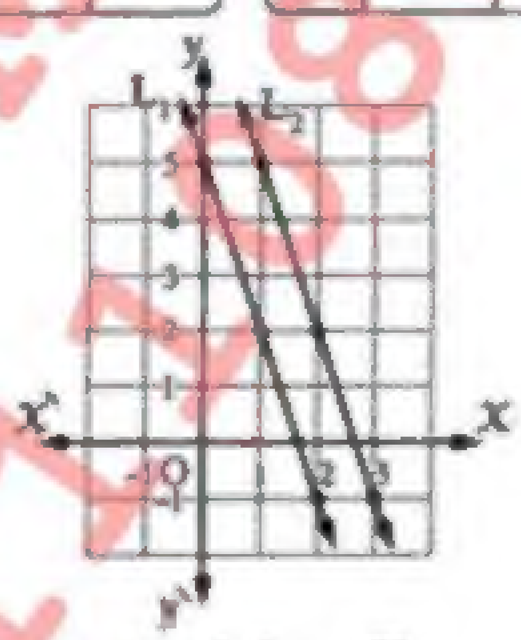
from the graph , the S.S. = $\{(3 , 2)\}$

3

$y = 5 - 3x$, $y = 8 - 3x$

x	0	1	2
y	5	2	−1

x	1	2	3
y	5	2	−1



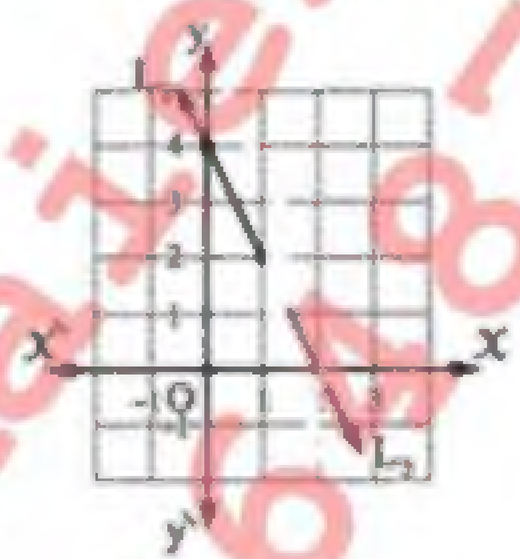
from the graph , the S.S. = $\varnothing$

4

$y = 4 - 2x$, $y = 4 - 2x$

x	0	1	2
y	4	2	0

x	0	1	2
y	4	2	0



from the graph ,

the S.S. = $\{(x , y) : y = 4 - 2x , (x , y) \in \mathbb{R} \times \mathbb{R}\}$

5

$y = -2x$, $x = 3 - 2y$

x	−1	0	1
y	2	0	−2

x	−1	1	3
y	2	1	0

Draw by yourself

from the graph , the S.S. = $\{(-1 , 2)\}$

6

$\because m_1 = -\frac{7}{4} , m_2 = \frac{-5}{-2} = \frac{5}{2}$ $\therefore m_1 \neq m_2$

$\therefore$ The two straight lines intersect at a point

$\therefore$ The number of solutions = 1

7

$\because m_1 = \frac{-9}{6} = \frac{-3}{2} , m_2 = \frac{-3}{2}$ $\therefore m_1 = m_2$

$\because$ The two straight lines intersect y-axis at the same point $(0 , 4)$

$\therefore$ The two straight lines are coincident

$\therefore$ The number of solutions is an infinite

8

$\because x + y = 6$ (1)

$, y - 2x = 0$ $\therefore y = 2x$ (2)

By substituting from (2) in (1) :

$\therefore x + 2x = 6$ $\therefore 3x = 6$ $\therefore x = 2$

By substituting in (2) : $\therefore y = 4$ $\therefore B (2 , 4)$

$\therefore$ The length of the altitude drawn from B to $\overleftrightarrow{AO}$ is 4 length units

$, \because A \in$ straight line L_1 $, A \in \overleftrightarrow{XX'}$

	at $y = 0$ in the equation $X + y = 6$ $\therefore X = 6$ $\quad\quad \therefore A(6 , 0)$ $\therefore AO = 6$ length units $\therefore$ *The area of* $\Delta ABO = \frac{1}{2} \times 6 \times 4 = 12$ square units.
B	Choose
1	B
2	C
3	D
4	A
5	D
6	A
7	A
8	D
9	C

Prep [3] - Second Term - Algebra - Unit [1] - Equations

Lesson [1] : Solving Two Equations Of First Degree In Two Variables

Second : Algebraically

Exercises

1. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x - y = 2$, $x + y = 4$ *(Red Sea 18)*

2. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x + 5y = 4$, $2x - 5y = 11$ *(Matrouh 18)*

3. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x = y + 4$, $3x + 4y = 5$ *(El-Dakahlia 18)*

4. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$2x - y = 3$, $x + 2y = 4$ *(El-Sharkia 19 , Alex. 18)*

5. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$3x + 2y = 4$, $x - 3y = 5$ *(Kafr El-Sheikh 19)*

6. Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$3x + 4y = 24$, $x - 2y = -2$ *(El-Gharbia 18 , Giza 12)*

7	**Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $3x - y = -4$, $y - 2x = 3$ *(Aswan 19)*
8	**Find algebraically in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x + 2y = 5$, $3x = y + 8$ *(El-Sharkia 18)*
9	**Find the values of a and b knowing that (3 , – 1) is the solution of the two equations :** $ax + by - 5 = 0$, $3ax + by = 17$ *(Luxor 18 , Damietta 17 , El-Gharbia 16)* « 2 , 1 »
10	**If (a , 2 b) is a solution for the two equations :** $3x - y = 5$ and $x + y = -1$, then find the values of a and b *(El-Dakahlia 17)* « 1 , – 1 »
11	If $f(x) = ax^2 + b$, $f(1) = 5$, $f(2) = 11$, then find the value of a and b *(El-Fayoum 09)* « 2 , 3 »
12	The sum of two natural numbers is 63 and their difference is 11 **Find the two numbers.** *(El-Beheira 16)* « 37 , 26 »
13	If three times a number is added to twice a second number the sum is 13 , and if the first number is added to three times the second number the sum is 16 , **find the two numbers.** *(Port Said 17)* « 1 , 5 »
14	A rectangle is with a length more than its width by 4 cm. If the perimeter of the rectangle is 28 cm. **Find the area of the rectangle.** *(El-Kalyoubia 19 , Cairo 17 , Alex. 12)* « 45 cm^2 »
15	Two acute angles in a right-angled triangle , the difference between their measures is 50° **Find the measure of each angle.** *(El-Beheira 19 , El-Kalyoubia 18 , Damietta 17)* « 70° , 20° »

16	🕮 A two-digit number , the sum of its digits is 11 If the two digits are reversed , then the resulted number is 27 more than the original number , what is the original number ? *(Kafr El-Sheikh 16)* « 47 »
17	A rectangle of perimeter 24 cm. If its length decreased by 4 cm. and its width increased by 2 cm. became a square. **Find the area of the square.** *(Ismailia 13)* « 25 cm^2 »

Solutions

A	ESSAY PROBLEMS
1	Adding the two equations we find that $2x = 6$ $\therefore x = 3$ Substituting in the second equation : $\therefore 3 + y = 4 \qquad \therefore y = 1$ $\therefore$ The S.S. $= \{(3 , 1)\}$
2	Adding the two equations we find that : $3x = 15$ $\therefore x = 5$ Substituting in the first equation : $\therefore 5 + 5y = 4 \qquad \therefore 5y = -1$ $\therefore y = \frac{-1}{5}$ $\therefore$ The S.S. $= \left\{\left(5 , \frac{-1}{5}\right)\right\}$
3	Substituting from the first equation in the second equation : $\therefore 3(y+4) + 4y = 5 \qquad \therefore 3y + 12 + 4y = 5$ $\therefore 7y = -7 \qquad \therefore y = -1$ Substituting in the first equation : $\therefore x = -1 + 4 \qquad \therefore x = 3$ $\therefore$ The S.S. $= \{(3 , -1)\}$
4	$\because 2x - y = 3$, multiplying by 2 $\therefore 4x - 2y = 6 \quad (1)$ $, \because x + 2y = 4 \quad (2)$ Adding (1) , (2) : $\therefore 5x = 10 \qquad \therefore x = 2$ Substituting in (2) : $\therefore 2 + 2y = 4 \qquad \therefore y = 1$ $\therefore$ The S.S. $= \{(2 , 1)\}$
5	$\because x - 3y = 5$, multiplying by -3 $\therefore -3x + 9y = -15 \quad (1)$ $, \because 3x + 2y = 4 \quad (2)$ Adding (1) , (2) : $\therefore 11y = -11 \qquad \therefore y = -1$ Substituting in (2) : $\therefore 3x - 2 = 4 \qquad \therefore x = 2$ $\therefore$ The S.S. $= \{(2 , -1)\}$
6	$\because 3x + 4y = 24 \quad (1)$ $, x - 2y = -2$, multiplying by 2 $\therefore 2x - 4y = -4 \quad (2)$ Adding (1) , (2) : $\therefore 5x = 20 \qquad \therefore x = 4$ Substituting in (1) : $\therefore 12 + 4y = 24 \qquad \therefore y = 3$ $\therefore$ The S.S. $= \{(4 , 3)\}$
7	Adding the two equations we find that : $x = -1$ Substituting in the second equation : $\therefore y + 2 = 3 \qquad \therefore y = 1$ $\therefore$ The S.S. $= \{(-1 , 1)\}$
8	From the second equation : $\because 3x = y + 8 \qquad \therefore y = 3x - 8 \quad (1)$ Substituting in the first equation : $\therefore x + 2(3x - 8) = 5 \qquad \therefore x + 6x - 16 = 5$ $\therefore 7x = 21 \qquad \therefore x = 3$ Substituting in (1) : $\therefore y = 3 \times 3 - 8$ $\therefore y = 1 \qquad \therefore$ The S.S. $= \{(3 , 1)\}$
9	$\because (3 , -1)$ is a solution for the equation $ax + by - 5 = 0 \qquad \therefore 3a - b = 5 \quad (1)$ $\because (3 , -1)$ is a solution for the equation $3ax + by = 17 \qquad \therefore 9a - b = 17$ $\therefore -9a + b = -17 \quad (2)$ Adding (1) and (2) : $\therefore -6a = -12 \qquad \therefore a = 2$ Substituting in (1) : $\therefore b = 1$
10	$\because (a , 2b)$ is a solution for the equation : $3x - y = 5$ $\therefore 3a - 2b = 5 \quad (1)$ $, \because (a , 2b)$ is a solution for the equation : $x + y = -1$ $\therefore a + 2b = -1 \quad (2)$ Adding (1) and (2) : $\therefore 4a = 4 \qquad \therefore a = 1$ Substituting in (1) : $\therefore b = -1$ **Another solution :** $\because 3x - y = 5 \quad (1) \qquad , x + y = -1 \quad (2)$ Adding (1) and (2) : $\therefore 4x = 4 \qquad \therefore x = 1$ Substituting in (2) : $\therefore y = -2$ $\therefore (1 , -2)$ is a solution for the two equations $, \because (a , 2b)$ is a solution for the two equations $\therefore (a , 2b) = (1 , -2) \qquad \therefore a = 1 \quad , \quad 2b = -2$ $\therefore b = -1$

11	$\because f(X) = aX^2 + b$, $f(1) = 5$ $\therefore a + b = 5$ (1) , $\because f(2) = 11$ $\therefore 4a + b = 11$ (2) Subtracting (1) from (2) : $\therefore 3a = 6$ $\therefore a = 2$ Substituting in (1) : $\therefore b = 3$
12	Let the two numbers be X and y $\therefore X + y = 63$ (1) , $X - y = 11$ (2) Adding (1) and (2) : $\therefore 2X = 74$ $\therefore X = 37$ Substituting in equ. (1) : $\therefore y = 26$ $\therefore$ The two numbers are 37 , 26
13	Let the first number be X , the second number be y $\therefore 3X + 2y = 13$ (1) , $X + 3y = 16$ (2) From (2) : $X = 16 - 3y$ (3) Substituting from (3) in (1) $\therefore 3(16 - 3y) + 2y = 13$ $\therefore 48 - 9y + 2y = 13$ $\therefore 48 - 7y = 13$ $\therefore 48 - 13 = 7y$ $\therefore 7y = 35$ $\therefore y = 5$ Substituting in (3) : $X = 1$ $\therefore$ The two numbers are 1 , 5
14	Let the length X cm. and the width be y cm $\therefore X - y = 4$ (1) , $2(X + y) = 28$ $\therefore X + y = 14$ (2) Adding (1) and (2) : $\therefore 2X = 18$ $\therefore X = 9$ Substituting in (1) : $\therefore y = 5$ $\therefore$ The length = 9 cm. , the width = 5 cm $\therefore$ The area of the rectangle = $9 \times 5 = 45$ cm^2
15	Let the measure of the first angle be $X°$ and let the measure of the second angle be $y°$ $\therefore X + y = 90$ (1) , $X - y = 50$ (2) Adding (1) and (2) : $\therefore 2X = 140$ $\therefore X = 70$ Substituting in (1) : $\therefore y = 20$ $\therefore$ The two measures are 70° , 20°
16	Let the units digit be X and the tens digit be y $\therefore X + y = 11$ (1) , $(y + 10X) - (X + 10y) = 27$ $9X - 9y = 27$ $\therefore X - y = 3$ (2) Adding (1) and (2) : $\therefore 2X = 14$ $X = 7$ Substituting in (1) $\therefore y = 4$ $\therefore$ The number is 47
17	Let the length of the rectangle be X cm. and the width be y cm $\therefore 2(X + y) = 24$ $\therefore X + y = 12$ (1) , $X - 4 = y + 2$ $\therefore X - y = 6$ (2) Adding (1) and (2) : $\therefore 2X = 18$ $\therefore X = 9$ $\therefore$ The side length of the square = $9 - 4 = 5$ cm $\therefore$ The area of the square = 25 cm^2

Prep [3] - Second Term - Algebra - Unit [1] - Equations

Lesson [2] : Solving An Equation Of Second Degree In One Unknown

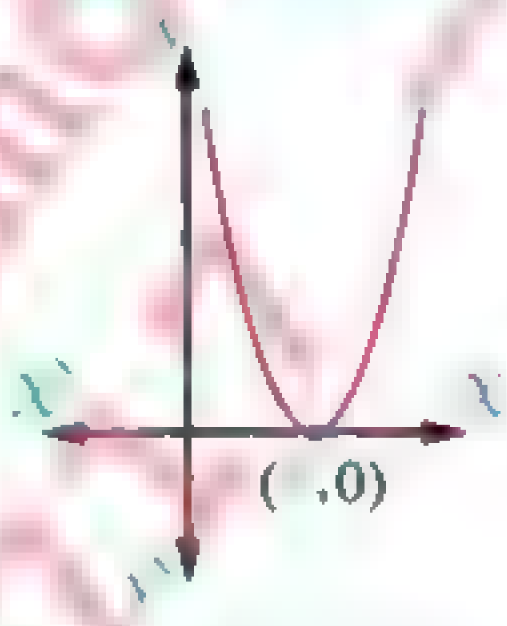
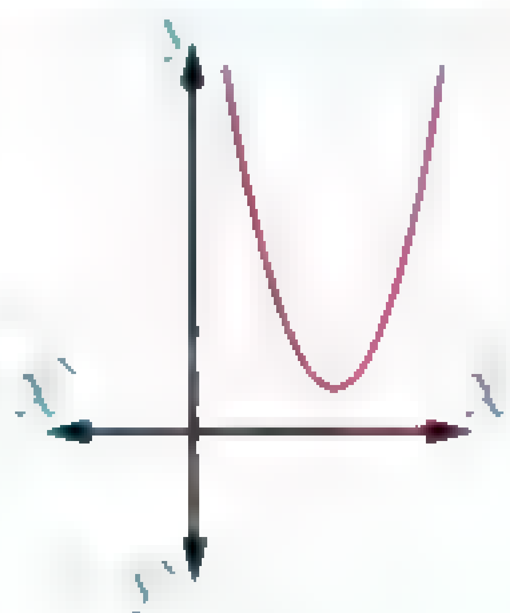
Part [1] : Graphically

First Solving an equation of the second degree in one unknown graphically

To solve an equation of the second degree in one unknown graphically , we do the following steps :

1. Put the equation in the form : $a x^2 + b x + c = 0$
2. Assume that : $f(x) = a x^2 + b x + c$, draw the curve of the function f
3. Determine the points of intersection of the function curve and x-axis , then the x-coordinates of these points of intersection are the solutions of the equation

According to that , we find three cases :

① The curve intersects x-axis at two points	② The curve touches x-axis at one point	③ The curve does not intersect x-axis
		
There are two solutions in $\mathbb{R}$ The S.S. = $\{\ell , m\}$	There is a unique solution in $\mathbb{R}$ The S.S. = $\{\ell\}$	There is no solution in $\mathbb{R}$ The S.S. = $\varnothing$

The following examples show the previous cases :

Example 1

Find graphically in $\mathbb{R}$ the S.S. of the equation :

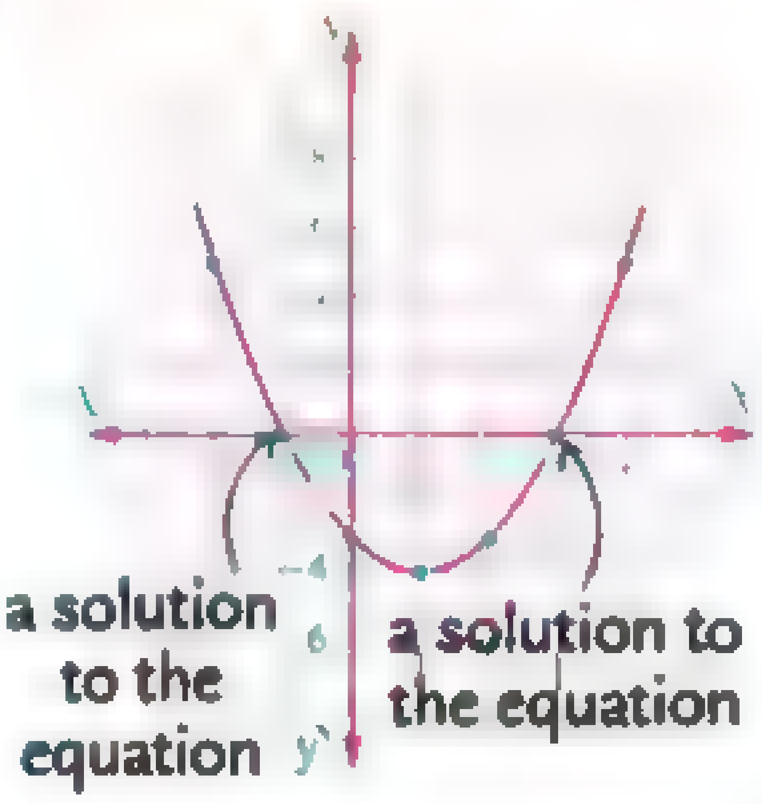
$$x^2 - 2x - 3 = 0$$

on the interval $[-2, 4]$

Solution

Let $f(x) = x^2 - 2x - 3$

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5



From the graph, the S.S. $= \{3, -1\}$

Example 2

Find graphically in $\mathbb{R}$ the S.S. of the equation :

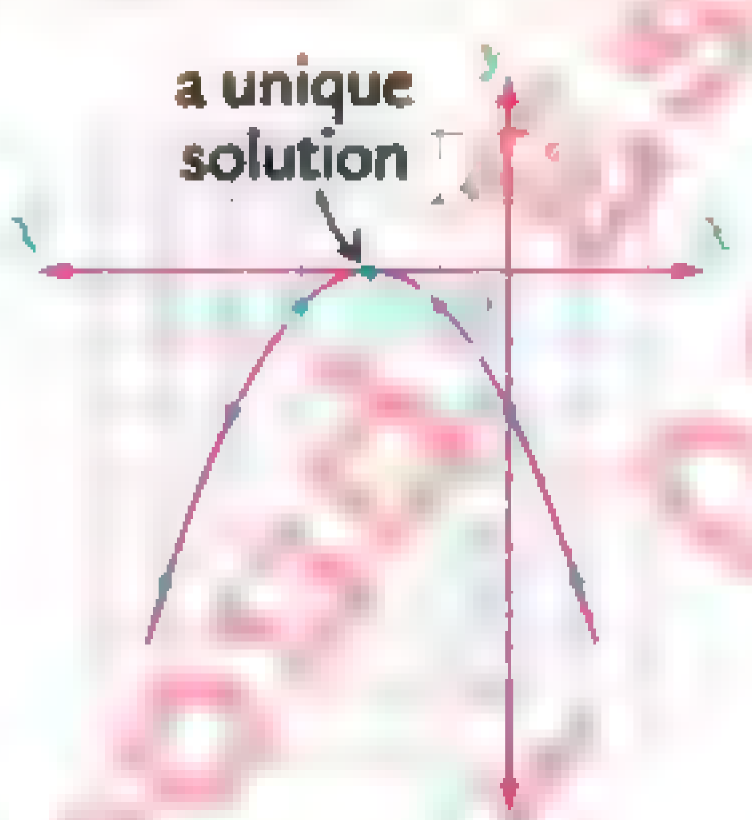
$$-x^2 - 4x - 4 = 0$$

on the interval $[-5, 1]$

Solution

Let $f(x) = -x^2 - 4x - 4$

x	-5	-4	-3	-2	-1	0	1
y	-9	-4	-1	0	-1	-4	-9



From the graph, the S.S. $= \{-2\}$

Example 3

Find graphically in $\mathbb{R}$ the S.S. of the equation :

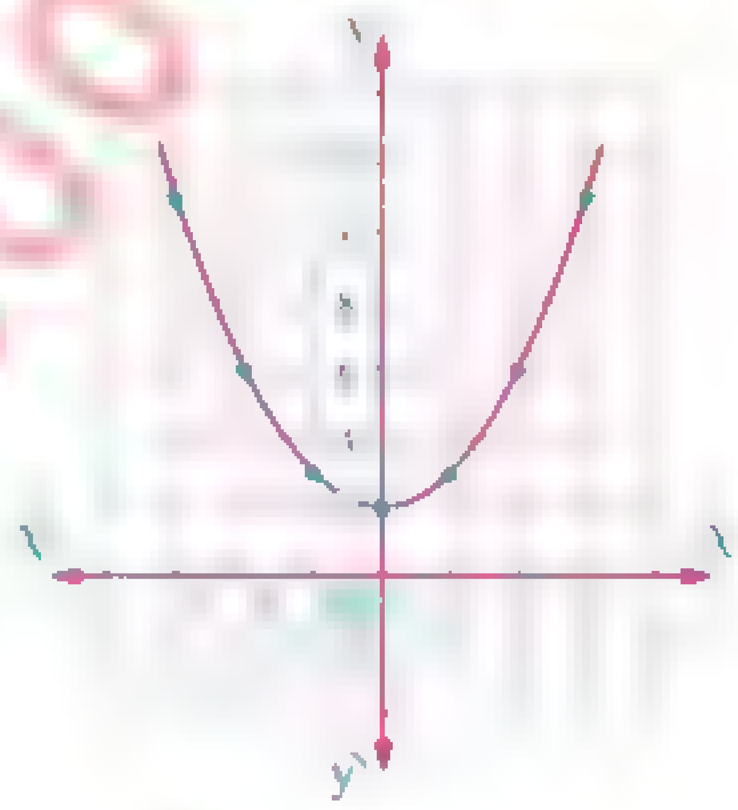
$$x^2 + 2 = 0$$

on the interval $[-3, 3]$

Solution

Let $f(x) = x^2 + 2$

x	-3	-2	-1	0	1	2	3
y	11	6	3	2	3	6	11



From the graph, the S.S. $= \varnothing$

" Remarks on the three previous examples

- **In example 1** :
  - * The vertex of the curve is : $(1, -4)$
  - * The minimum value $= -4$
  - * The equation of the axis of symmetry of the curve is : $x = 1$
- **In example 2** :
  - * The vertex of the curve is : $(-2, 0)$
  - * The maximum value $= 0$
  - * The equation of the axis of symmetry of the curve is : $x = -2$
- **In example 3** :
  - * The vertex of the curve is : $(0, 2)$
  - * The minimum value $= 2$
  - * The equation of the axis of symmetry of the curve is : $x = 0$ "

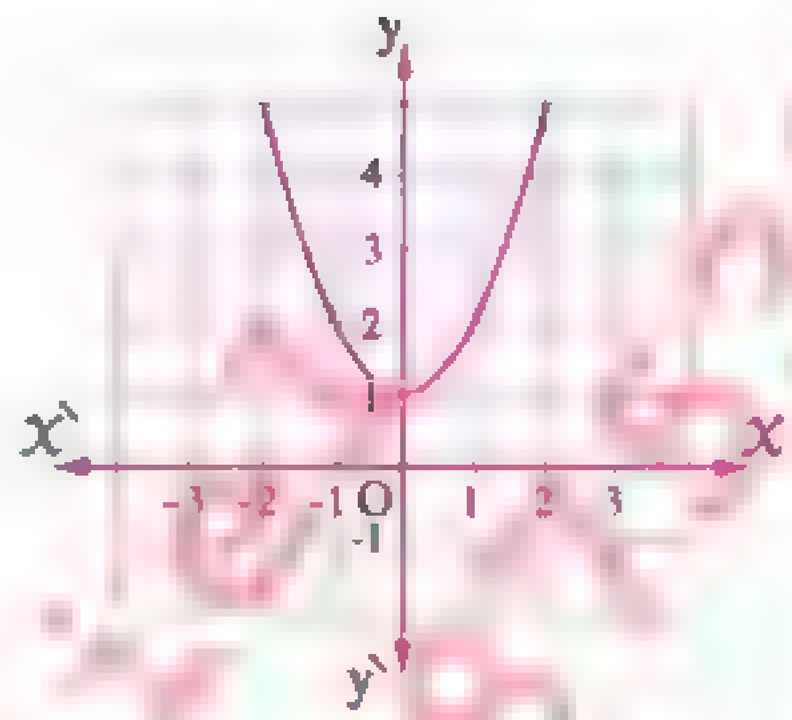
Exercises

[A] Essay problems : -

1	**Draw the graphical representation of the function f in the given interval , then find the solution set of the equation $f(X) = 0$:** $f(X) = 2X^2 + 5X$ in the interval $[-4, 2]$ *(Souhag 13)*
2	Represent graphically the function $f : f(X) = X^2 - 2X$ in the interval $[-1, 3]$, from the graph find the S.S. of the equation : $X^2 - 2X = 0$ *(Suez 12)*
3	**Graph the function f :** $f(X) = X^2 - 4X + 3$ on the interval $[-1, 5]$ **and from the graph , find :** 1 The minimum value of the function. 2 The equation of the axis of symmetry. 3 The S.S. of the equation $f(X) = 0$ *(El-Monofia 12)*
4	**Draw a graphical representation of the function f where $f(X) = 6X - X^2 - 9$ in the interval $[0, 5]$ and from the drawing find :** 1 The maximum value or the minimum value of the function. 2 The solution set of the equation : $6X - X^2 - 9 = 0$ *(Port Said 12)*
5	**Draw the graphical representation of the function f in the given interval , then find the solution set of the equation $f(X) = 0$:** $f(X) = X(X - 5) + 3$ in the interval $[0, 5]$ *(El-Monofia 11)*

[B] Choose the correct : -

1 The opposite figure represents the curve of a quadratic function f , then the solution set of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(Cairo 16)*

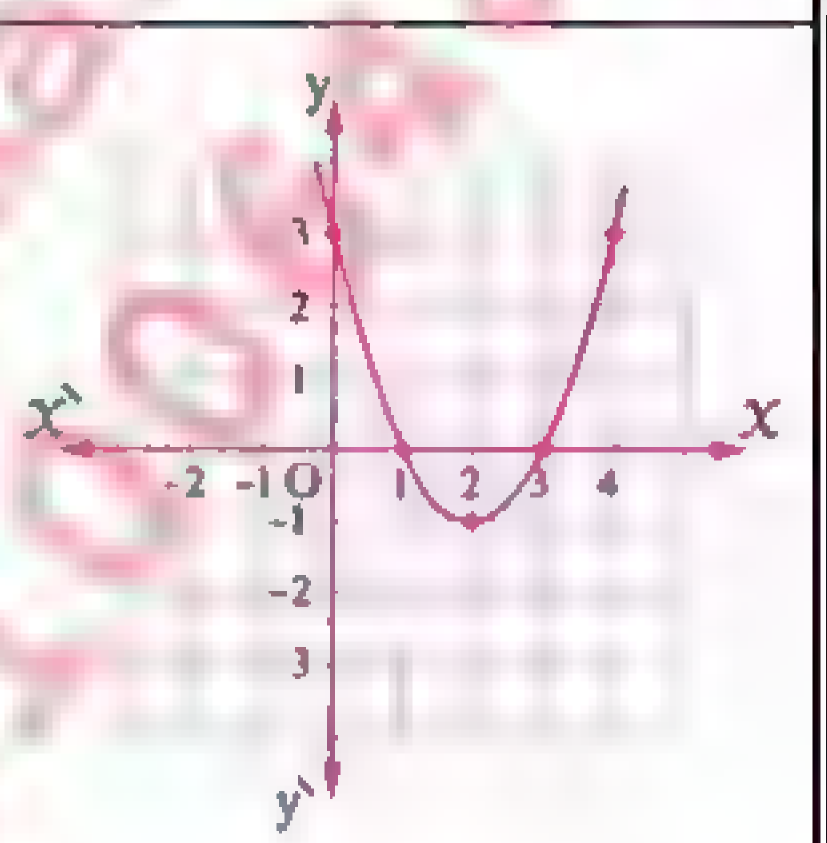


(a) $\varnothing$ (b) $\{1\}$

(c) $\{0\}$ (d) $\{(0 , 1)\}$

2 **In the opposite figure :**

The S.S. of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(Cairo 15)*



(a) $(2 , -1)$ (b) $\{(3 , 1)\}$

(c) $\{3 , 1\}$ (d) $(3 , 0)$

3 If the curve of the quadratic function does not intersect the X-axis at any point , then the number of solutions of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(El-Monofia 17 , Qena 04)*

(a) a unique solution. (b) two solutions.

(c) an infinite number. (d) zero.

4 If the curve of the quadratic function f passes through the points $(-1 , 0)$, $(0 , -4)$, $(4 , 0)$ and $(0 , -6)$, then the solution set of the equation $f(X) = 0$ in $\mathbb{R}$ is .... *(El-Gharbia 19)*

(a) $\{-1 , 0\}$ (b) $\{-4 , 0\}$ (c) $\{-1 , 4\}$ (d) $\{4 , -4\}$

5 If $X = 3$ is one of the solutions of the equation : $X^2 - aX - 6 = 0$, then a = ... *(Suez 17)*

(a) 3 (b) 2 (c) 1 (d) -1

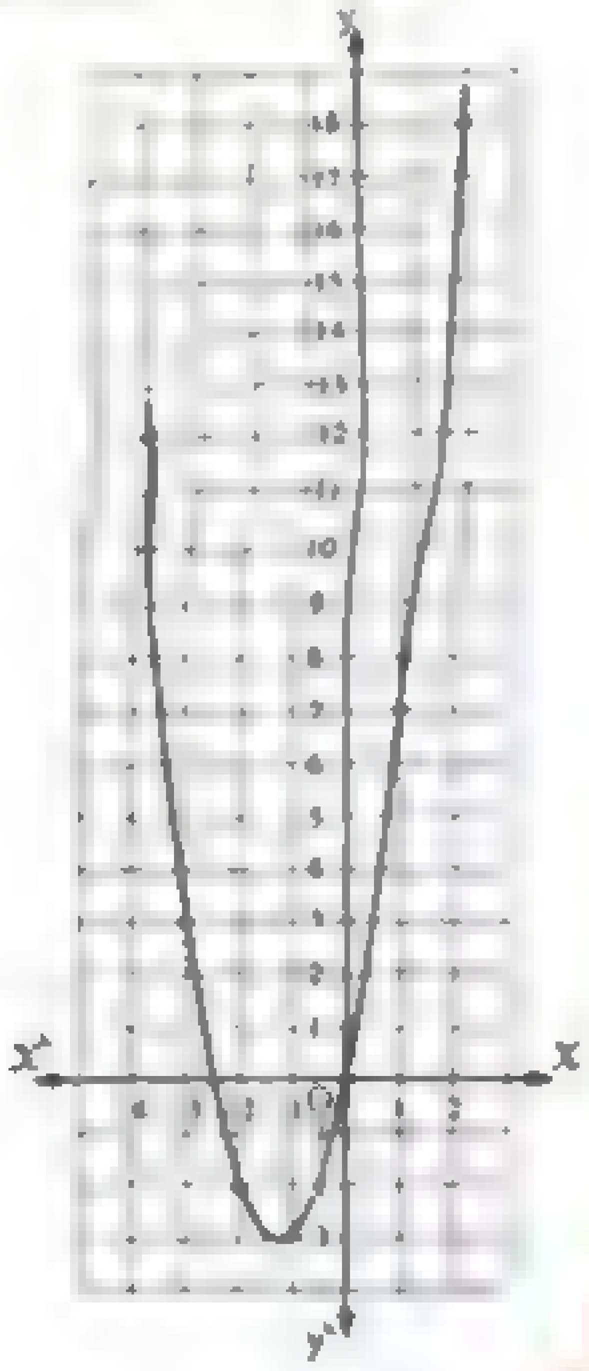
Solutions

A Essay Problems

1

$f(x) = 2x^2 + 5x$

x	-4	-3	-2	-1	0	1	2
y	12	3	-2	-3	0	7	18



From the graph : The S.S. = $\{-2.5 , 0\}$

2

$f(x) = x^2 - 2x$

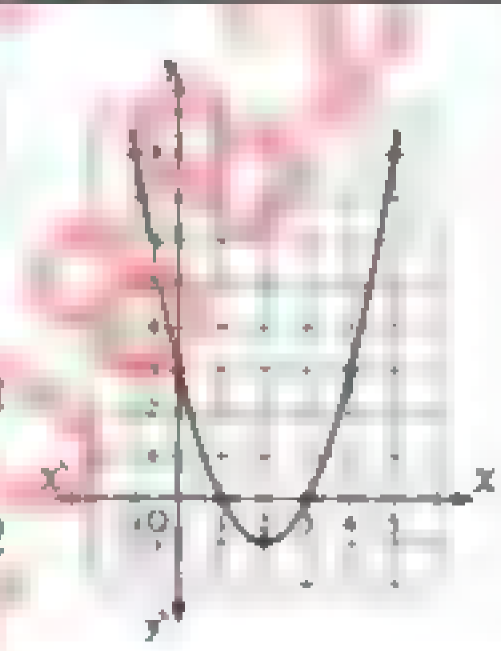
x	-1	0	1	2	3
y	3	0	-1	0	3

From the graph : The S.S. = $\{0 , 2\}$

3

$f(x) = x^2 - 4x + 3$

x	1	0	1	2	3	4	5
y	8	3	0	-1	0	3	8



From the graph

1. The minimum value = -1
2. The equation of the axis of symmetry is $x = 2$
3. The S.S. = $\{1 , 3\}$

4

$f(x) = 6x - x^2 - 9$

x	0	1	2	3	4	5
y	-9	-4	-1	0	1	4

From the graph .

1. The maximum value = 0
2. The S.S. = $\{3\}$

5

$f(x) = x^2 - 5x + 3$

x	0	1	2	3	4	5
y	3	-1	-3	-3	-1	3

Draw by yourself and from the graph

The S.S. = $\{0.7 , 4.3\}$ approximately.

A Choose

1	A
2	C
3	D
4	C
5	C

Prep [3] - Second Term - Algebra - Unit [1] - Equations

Lesson [2] : Solving An Equation Of Second Degree In One Unknown

Part [2] : Algebraically

Second Solving an equation of the second degree in one unknown using the general rule (general formula)

The general rule (general formula) for solving an equation of the second degree in one unknown :

If $a x^2 + b x + c = 0$ where a , b and c are real numbers , $a \neq 0$

, then $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

i.e. The solution set of the equation $= \left\{ \frac{-b + \sqrt{b^2 - 4ac}}{2a} , \frac{-b - \sqrt{b^2 - 4ac}}{2a} \right\}$

“ **Remarks on the previous example**

- In 1 : The value of : $b^2 - 4ac = 49 > 0$ and the equation had two solutions which are : 6 and -1
  Generally if : $b^2 - 4ac > 0$, then the equation has **two different solutions** in $\mathbb{R}$
- In 2 : The value of : $b^2 - 4ac = 0$ and the equation had one solution which is : $\frac{1}{2}$
  Generally if : $b^2 - 4ac = 0$, then the equation has **a unique solution** in $\mathbb{R}$
- In 3 : The value of : $b^2 - 4ac = -4 < 0$ and the equation had no real solutions
  Generally if : $b^2 - 4ac < 0$, then the equation has **no real solutions** in $\mathbb{R}$ ”

Exercises

[A] Essay problems : -

1	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2+7x+2=0$ approximating the result to the nearest tenth. *(El-Kalyoubia 16)*
2	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2-4x+1=0$ approximating the result to the nearest two decimal digits. *(Giza 17 ، Aswan 14 ، Alexandria 13)*
3	**Find in ℝ the S.S. of each of the following equations using the general formula :** $2x^2-4x+1=0$ rounding the result to three decimal digits. *(El-Dakahlia 19 ، Qena 12)*
4	**Find in ℝ the S.S. of each of the following equations using the general formula :** $3x^2-6x+1=0$ rounding the result to the nearest three decimals. *(South Sinai 18)*
5	**Find in ℝ the S.S. of each of the following equations using the general formula :** $2x^2+5x=0$ *(Alexandria 19)*
6	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2+3x+5=0$ *(El-Fayoum 19)*
7	**Find in ℝ the S.S. of each of the following equations using the general formula :** $x^2+8x+9=0$ ، where $\sqrt{7}\approx 2.65$ *(Ismailia 09)*
8	**Find in ℝ the S.S. of each of the following equations using the general formula :** $2x^2-x-2=0$ ، where $\sqrt{17}\approx 4.12$ *(Luxor 19)*
9	**Find in ℝ the solution set of each of the following equations using the general formula approximating the result to three decimal digits :** $2x^2-10x=1$ *(Damietta 13)*

10	Find in $\mathbb{R}$ the solution set of each of the following equations using the general formula approximating the result to three decimal digits : $x(x-1)=4$ *(Souhag 19)*
11	Find in $\mathbb{R}$ the solution set of each of the following equations using the general formula approximating the result to three decimal digits : $x+\frac{4}{x}=6$ *(Damietta 19)*
12	Find in $\mathbb{R}$ the solution set of each of the following equations using the general formula approximating the result to three decimal digits : $\frac{8}{x^2}+\frac{1}{x}=1$ *(El-Fayoum 12)*

[B] Choose the correct :

1	If $x=3$ is one of the solutions of the equation : $x^2-ax-6=0$, then a = ……… *(Suez 17)* (a) 3 (b) 2 (c) 1 (d) – 1	C
2	In the equation : $ax^2+bx+c=0$, if $b^2-4ac>0$, then this equation has ……… roots in $\mathbb{R}$ *(El-Fayoum 19 , Damietta 16)* (a) 1 (b) 2 (c) zero (d) an infinite number	B

Solutions

A	Essay Problems
1	$\because a=1 , b=7 , c=2$ $\therefore x=\frac{-7\pm\sqrt{49-8}}{2}=\frac{-7\pm\sqrt{41}}{2}$ $\therefore x\approx-0.3$ or $x\approx-6.7$ $\therefore$ The S.S. $=\{-0.3 , -6.7\}$
2	$\because a=1 , b=-4 , c=1$ $\therefore x=\frac{4\pm\sqrt{16-4}}{2}=\frac{4\pm\sqrt{12}}{2}$ $\therefore x\approx0.27$ or $x\approx3.73$ $\therefore$ The S.S. $=\{0.27 , 3.73\}$
3	$\because a=2 , b=-4 , c=1$ $\therefore x=\frac{4\pm\sqrt{16-8}}{4}=\frac{4\pm\sqrt{8}}{4}$ $\therefore x\approx0.293$ or $x\approx1.707$ $\therefore$ The S.S. $=\{0.293 , 1.707\}$
4	$\because a=3 , b=-6 , c=1$ $\therefore x=\frac{6\pm\sqrt{36-12}}{6}=\frac{6\pm\sqrt{24}}{6}$ $\therefore x\approx0.184$ or $x\approx1.816$ $\therefore$ The S.S. $=\{0.184 , 1.816\}$
5	$\because a=2 , b=5 , c=0$ $\therefore x=\frac{-5\pm\sqrt{25-0}}{4}=\frac{-5\pm5}{4}$ $\therefore x=\frac{0}{4}=0$ or $x=\frac{-10}{4}=-2.5$ $\therefore$ The S.S. $=\{0 , -2.5\}$
6	$\because a=1 , b=3 , c=5$ $\therefore x=\frac{-3\pm\sqrt{9-20}}{2}=\frac{-3\pm\sqrt{-11}}{2}$ $\therefore$ The S.S. $=\varnothing$
7	$\because a=1 , b=8 , c=9$ $\therefore x=\frac{-8\pm\sqrt{64-36}}{2}=\frac{-8\pm2\sqrt{7}}{2}$ $=-4\pm\sqrt{7}=-4\pm2.65$ $\therefore$ The S.S. $=\{-1.35 , -6.65\}$
8	$\because a=2 , b=-1 , c=-2$ $\therefore x=\frac{1\pm\sqrt{1+16}}{4}=\frac{1\pm\sqrt{17}}{4}=\frac{1\pm4.12}{4}$ $\therefore$ The S.S. $=\{-0.78 , 1.28\}$
9	$\because 2x^2-10x-1=0$ $\therefore a=2 , b=-10 , c=-1$ $\therefore x=\frac{10\pm\sqrt{100+8}}{4}=\frac{10\pm\sqrt{108}}{4}=\frac{10\pm6\sqrt{3}}{4}$ $=\frac{5\pm3\sqrt{3}}{2}$ $\therefore x\approx5.098$ or $x\approx-0.098$ $\therefore$ The S.S. $=\{-0.098 , 5.098\}$
10	$\because x^2-x-4=0$ $\therefore a=1 , b=-1 , c=-4$ $\therefore x=\frac{1\pm\sqrt{1+16}}{2}=\frac{1\pm\sqrt{17}}{2}$ $\therefore x\approx2.562$ or $x\approx-1.562$ $\therefore$ The S.S. $=\{2.562 , -1.562\}$
11	Multiplying the equation by x : $\therefore x^2+4=6x$ $\quad\therefore x^2-6x+4=0$ $\therefore a=1 , b=-6 , c=4$ $\therefore x=\frac{6\pm\sqrt{36-16}}{2}=\frac{6\pm\sqrt{20}}{2}$ $\therefore x\approx5.236$ or $x\approx0.764$ $\therefore$ The S.S. $=\{5.236 , 0.764\}$

12	Multiplying the equation by x^2 : $\therefore 8 + x = x^2 \quad \therefore x^2 - x - 8 = 0$ $\therefore a = 1 , b = -1 , c = -8$ $\therefore x = \frac{1 \pm \sqrt{1+32}}{2} = \frac{1 \pm \sqrt{33}}{2}$ $\therefore x = 3.372$ or $x = -2.372$ $\therefore$ The S.S. $= \{3.372 , -2.372\}$
B	Choose
1	C
2	B

Lesson [3] : Solving Two Equations In Two Variables, One Is Of The First Degree And The Other Is Of The Second Degree

Part [1] : -

Exercises

[A] Essay problems : -

1 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x = y$, $x^2 + y^2 = 2$ *(Souhag 09)*

2 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x - 3 = 0$, $x^2 + y^2 = 25$ *(Cairo 19)*

3 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x - 2y = 0$, $x^2 - y^2 = 3$ *(Port Said 17)*

4 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x - y = 0$, $x^2 + xy + y^2 = 27$ *(Alex. 19)*

5 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$y - 2x = 0$, $xy = 18$ *(El-Sharkia 14)*

6 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :
$x + y = 0$, $y^2 = x$ *(6^{th} October 11)*

7 Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :

	$x - y = 0$, $x = \frac{4}{y}$ *(El-Dakahlia 19 , Ismailia 18)*
8	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $y = x - 1$, $y^2 + x = 7$ *(Qena 09)*
9	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x = 5 - y$, $x^2 - y^2 = 55$ *(Matrouh 08)*
10	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x - y = 1$, $x^2 + y^2 = 25$ *(Aswan 19 , Port said 18)*
11	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x + y = 7$, $y^2 - x^2 = 7$ *(Kafr El-Sheikh 15)*
12	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x - y - 2 = 0$, $x^2 - y^2 = 0$ *(El-Kalyoubia 09)*
13	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $2x + y = 10$, $x^2 + y^2 = 25$ *(El-Kalyoubia 05)*
14	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $y - x = 3$, $x^2 - 2x + 3y = 15$ *(Alex. 11)*
15	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x + y = 7$, $xy = 12$ *(Qena 17)*

16	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x + y = 5$, $\frac{xy}{6} = 1$ *(Monofia 08)*
17	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $y - x = 2$, $x^2 + xy - 4 = 0$ *(El-Beheira 19)*
18	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x - 2y - 1 = 0$, $x^2 - xy = 0$ *(Kafer El-Sheikh 19)*
19	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $x + y = 1$, $x^2 + xy + y^2 = 3$ *(South Sinai 18)*
20	**Find in $\mathbb{R} \times \mathbb{R}$ the S.S. of each pair of the following equations :** $y - x = 3$, $x^2 + y^2 - xy = 13$ *(El-Kalyoubia 17)*
21	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :** $x = 0$, $x^2 + y^2 + 4x + 3y - 10 = 0$ *(Ismailia 03)*
22	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :** $x - 2y = 8$, $y^2 = x$ *(Dametta 09)*
23	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :** $x + 2y = 2$, $x^2 + 2xy = 2$ *(El-Sharkia 19)*
24	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :**

	$x + y = 2$, $x^2 + y^2 + 2xy + y = 6$ (New Valley 13)
25	**Find in $\mathbb{R} \times \mathbb{R}$ the solution set of each pair of the following equations :** $x + y = 2$, $\frac{1}{x} + \frac{1}{y} = 2$, where $x \neq 0$, $y \neq 0$ (El-Menia 19)

[B] Choose the correct : -

1	The S.S. of the two equations : $x - y = 0$, $xy = 9$ in $\mathbb{R} \times \mathbb{R}$ is .... (Qena 18 , El-Gharbia 11) (a) $\{(0 , 0)\}$ (b) $\{(-3 , 3)\}$ (c) $\{(3 , 3)\}$ (d) $\{(-3 , -3) , (3 , 3)\}$
2	The S.S. of the two equations : $x + y = 0$, $x^2 + y^2 = 2$ in $\mathbb{R} \times \mathbb{R}$ is .... (Assiut 13) (a) $\{(0 , 0)\}$ (b) $\{(1 , -1)\}$ (c) $\{(-1 , 1)\}$ (d) $\{(1 , -1) , (-1 , 1)\}$
3	The ordered pair which satisfies each of the two equations : $xy = 2$, $x - y = 1$ is .... (El-Sharkia 12) (a) $(1 , 1)$ (b) $(2 , 1)$ (c) $(1 , 2)$ (d) $(\frac{1}{2} , 1)$
4	One of the solutions for the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is .... (El - Kalyoubia 19 , Qena 17 , Port Said 14) (a) $(-4 , 2)$ (b) $(2 , -4)$ (c) $(3 , 1)$ (d) $(4 , 2)$
5	If $y = 1 - x$, $(x + y)^2 + y = 5$, then $y =$ .... (El-Fayoum 12) (a) 5 (b) 3 (c) -4 (d) 4
6	If $x^2 + xy = 15$, $x + y = 5$, then $x =$ .... (Cairo 06) (a) 3 (b) 4 (c) 5 (d) 6

Solutions

A Essay Problems

1

Substituting from equ. (1) in equ. (2) :

$\therefore x^2 + x^2 = 2$ $\therefore 2x^2 = 2$

$\therefore x^2 = 1$ $\therefore x = 1$ or $x = -1$

$\therefore y = 1$ or $y = -1$

The S.S. $= \{(1 , 1) , (-1 , -1)\}$

2

$\because x - 3 = 0$ $\therefore x = 3$

Substituting in second equation :

$\therefore 9 + y^2 = 25$ $\therefore y^2 = 16$

$\therefore y = 4$ or $y = -4$

$\therefore$ The S.S. $= \{(3 , 4) , (3 , -4)\}$

3

$\because x - 2y = 0$ $\therefore x = 2y$ (1)

Substituting in the other equation

$\therefore (2y)^2 - y^2 = 3$ $\therefore 4y^2 - y^2 = 3$

$\therefore 3y^2 = 3$ $\therefore y^2 = 1$

$\therefore y = 1$ or $y = -1$

From (1) : $\therefore x = 2$ or $x = -2$

$\therefore$ The S.S. $= \{(2 , 1) , (-2 , -1)\}$

4

$\because x - y = 0$ $\therefore x = y$ (1)

Substituting in the other equation :

$\therefore x^2 + x \times x + x^2 = 27$

$\therefore 3x^2 = 27$ $\therefore x^2 = 9$

$\therefore x = 3$ or $x = -3$

From (1) : $\therefore y = 3$ or $y = -3$

$\therefore$ The S.S. $= \{(3 , 3) , (-3 , -3)\}$

5

$\because y - 2x = 0$ $\therefore y = 2x$ (1)

Substituting in the other equation :

$\therefore x(2x) = 18$ $\therefore 2x^2 = 18$ $\therefore x^2 = 9$

$\therefore x = 3$ or $x = -3$

From (1) : $\therefore y = 6$ or $y = -6$

$\therefore$ The S.S. $= \{(3 , 6) , (-3 , -6)\}$

6

Substituting from equ. (2) in equ. (1) :

$\therefore y^2 + y = 0$ $\therefore y(y + 1) = 0$

$\therefore y = 0$ or $y = -1$

Substituting in equ. (1) : $\therefore x = 0$ or $x = 1$

$\therefore$ The S.S. $= \{(0 , 0) , (1 , -1)\}$

7

$\because x - y = 0$ $\therefore x = y$ (1)

substituting in the second equation :

$\therefore y = \frac{4}{y}$ $\therefore y^2 = 4$

$\therefore y = 2$ or $y = -2$

From (1) : $\therefore x = 2$ or $x = -2$

$\therefore$ The S.S. $= \{(2 , 2) , (-2 , -2)\}$

8

Substituting from equ. (1) in equ. (2) :

$\therefore (x - 1)^2 + x = 7$

$\therefore x^2 - 2x + 1 + x - 7 = 0$

$\therefore x^2 - x - 6 = 0$ $\therefore (x - 3)(x + 2) = 0$

$\therefore x = 3$ or $x = -2$

Substituting in equ. (1) : $\therefore y = 2$ or $y = -3$

$\therefore$ The S.S. $= \{(3 , 2) , (-2 , -3)\}$

9

Substituting from equ. (1) in equ. (2) :

$\therefore (5 - y)^2 - y^2 = 55$

$\therefore 25 - 10y + y^2 - y^2 = 55$

$\therefore -10y = 30$ $\therefore y = -3$

Substituting in equ. (1) : $\therefore x = 8$

$\therefore$ The S.S. $= \{(8 , -3)\}$

10

$\because x - y = 1$ $\therefore x = 1 + y$ (1)

Substituting in the second equation

$\therefore (1 + y)^2 + y^2 = 25$

$\therefore 1 + 2y + y^2 + y^2 = 25$ $\therefore 2y^2 + 2y - 24 = 0$

$\therefore y^2 + y - 12 = 0$ $\therefore (y + 4)(y - 3) = 0$

$\therefore y = -4$ or $y = 3$

And from (1) : $\therefore x = -3$ or $x = 4$

$\therefore$ The S.S. $= \{(-3 , -4) , (4 , 3)\}$

11	$\because x + y = 7 \qquad \therefore y = 7 - x \qquad (1)$ Substituting in the other equation : $\therefore (7 - x)^2 - x^2 = 7$ $\therefore 49 - 14x + x^2 - x^2 = 7$ $\therefore -14x = -42 \qquad \therefore x = 3$ From (1) : $\therefore y = 4$ $\therefore$ The S.S. $= \{(3, 4)\}$
12	$\because x - y - 2 = 0 \qquad \therefore x = y + 2$ Substituting in the second equation : $\therefore (y + 2)^2 - y^2 = 0$ $\therefore y^2 + 4y + 4 - y^2 = 0$ $\therefore 4y = -4 \qquad \therefore y = -1$ From (1) : $\therefore x = 1$ $\therefore$ The S.S. $= \{(1, -1)\}$
13	$\because 2x + y = 10 \qquad \therefore y = 10 - 2x \qquad (1)$ Substituting in the second equation : $\therefore x^2 + (10 - 2x)^2 = 25$ $\therefore x^2 + 100 - 40x + 4x^2 - 25 = 0$ $\therefore 5x^2 - 40x + 75 = 0 \qquad \therefore x^2 - 8x + 15 = 0$ $\therefore (x - 3)(x - 5) = 0 \qquad \therefore x = 3$ or $x = 5$ From (1) : $\therefore y = 4$ or $y = 0$ $\therefore$ The S.S. $= \{(3, 4), (5, 0)\}$
14	$\because y - x = 3 \qquad \therefore y = x + 3 \qquad (1)$ Substituting in the equ. (2) : $\therefore x^2 - 2x + 3(x + 3) = 15$ $\therefore x^2 - 2x + 3x + 9 - 15 = 0$ $\therefore x^2 + x - 6 = 0 \qquad \therefore (x + 3)(x - 2) = 0$ $\therefore x = -3$ or $x = 2$ From (1) : $\therefore y = 0$ or $y = 5$ $\therefore$ The S.S. $= \{(-3, 0), (2, 5)\}$
15	$\because x + y = 7 \qquad \therefore y = 7 - x \qquad (1)$ Substituting in the second equation : $\therefore x(7 - x) = 12 \qquad \therefore 7x - x^2 = 12$ $\therefore x^2 - 7x + 12 = 0 \qquad \therefore (x - 3)(x - 4) = 0$ $\therefore x = 3$ or $x = 4$ From (1) : $\therefore y = 4$ or $y = 3$ $\therefore$ The S.S. $= \{(3, 4), (4, 3)\}$
16	$\because x + y = 5 \qquad \therefore y = 5 - x \qquad (1)$ $\because \frac{x y}{6} = 1 \qquad \therefore x y = 6 \qquad (2)$ Substituting from (1) in (2) : $\therefore x(5 - x) = 6 \qquad \therefore 5x - x^2 - 6 = 0$ $\therefore x^2 - 5x + 6 = 0 \qquad \therefore (x - 2)(x - 3) = 0$ $\therefore x = 2$ or $x = 3$ And from (1) $\therefore y = 3$ or $y = 2$ $\therefore$ The S.S. $= \{(2, 3), (3, 2)\}$
17	$\because y - x = 2 \qquad \therefore y = x + 2 \qquad (1)$ Substituting in the second equation : $\therefore x^2 + x(x + 2) - 4 = 0$ $\therefore x^2 + x^2 + 2x - 4 = 0$ $\therefore 2x^2 + 2x - 4 = 0 \qquad \therefore x^2 + x - 2 = 0$ $\therefore (x + 2)(x - 1) = 0 \qquad \therefore x = -2$ or $x = 1$ From (1) : $\therefore y = 0$ or $y = 3$ $\therefore$ The S.S. $= \{(-2, 0), (1, 3)\}$
18	$\because x - 2y - 1 = 0 \qquad \therefore x = 2y + 1$ Substituting in the second equation : $\therefore (2y + 1)^2 - y(2y + 1) = 0$ $\therefore 4y^2 + 4y + 1 - 2y^2 - y = 0$ $\therefore 2y^2 + 3y + 1 = 0 \qquad \therefore (2y + 1)(y + 1) = 0$ $\therefore y = -\frac{1}{2}$ or $y = -1$ From (1) : $\therefore x = 0$ or $x = -1$ $\therefore$ The S.S. $= \{(0, -\frac{1}{2}), (-1, -1)\}$

19	$\because x+y=1 \quad \therefore y=1-x \quad (1)$ Substituting in the second equation : $\therefore x^2+x(1-x)+(1-x)^2=3$ $\therefore x^2+x-x^2+1-2x+x^2-3=0$ $\therefore x^2-x-2=0$ $\therefore (x-2)(x+1)=0 \quad \therefore x=2$ or $x=-1$ From (1) : $\therefore y=-1$ or $y=2$ $\therefore$ The S.S. $=\{(2,-1),(-1,2)\}$
20	$\because y-x=3 \quad \therefore y=3+x \quad (1)$ Substituting in the second equation : $\therefore x^2+(3+x)^2-x(3+x)=13$ $\therefore x^2+9+6x+x^2-3x-x^2-13=0$ $\therefore x^2+3x-4=0 \quad \therefore (x-1)(x+4)=0$ $\therefore x=1$ or $x=-4$ And from (1) : $\therefore y=4$ or $y=-1$ $\therefore$ The S.S. $=\{(1,4),(-4,-1)\}$
21	Substituting from equ. (1) in equ. (2) : $\therefore y^2+3y-10=0 \quad \therefore (y-2)(y+5)=0$ $\therefore y=2$ or $y=-5$ $\therefore$ The S.S. $=\{(0,2),(0,-5)\}$
22	Substituting from equ. (2) in equ. (1) : $\therefore y^2-2y=8 \quad \therefore y^2-2y-8=0$ $\therefore (y+2)(y-4)=0 \quad \therefore y=-2$ or $y=4$ Substituting in equ. (1) : $\therefore x=4$ or $x=16$ $\therefore$ The S.S. $=\{(4,-2),(16,4)\}$
23	$\because x^2+2xy=2 \quad \therefore x(x+2y)=2$ $,\because x+2y=2 \quad \therefore 2x=2 \quad \therefore x=1$ Substituting in the first equation : $\therefore 1+2y=2 \quad \therefore y=\frac{1}{2}$ $\therefore$ The S.S. $=\{(1,\frac{1}{2})\}$

24	$\because x^2+2xy+y^2+y=6$ $\therefore (x+y)^2+y=6 \quad ,\because x+y=2$ $\therefore 2^2+y=6 \quad \therefore y=2$ Substituting in the first equation : $\therefore x+2=2 \quad \therefore x=0$ $\therefore$ The S.S. $=\{(0,2)\}$
25	$\because x+y=2 \quad \therefore x=2-y \quad (1)$ $\because \frac{1}{x}+\frac{1}{y}=2 \quad \therefore y+x=2xy$ $\therefore y+x-2xy=0 \quad (2)$ Substituting from (1) in (2) : $\therefore y+2-y-2y(2-y)=0$ $\therefore 2y^2-4y+2=0$ $\therefore y^2-2y+1=0 \quad \therefore (y-1)^2=0 \quad \therefore y=1$ From (1) : $\therefore x=1$ $\therefore$ The S.S. $=\{(1,1)\}$

B	Choose
1	D
2	D
3	B
4	D
5	D
6	A

Lesson [3] : Solving Two Equations In Two Variables, One Is Of The First Degree And The Other Is Of The Second Degree

Part [2] : -

Applications on solving two equations in two variables one of them of the first degree and the other of the second degree :

Exercises

[C] Essay problems : -

1	The sum of two real positive numbers is 17 and their product is 72 **Find the two numbers.** *(Alex. 09)*
2	The sum of two real numbers is 9 and the difference between their squares equals 45 **Find the two numbers.** *(El-Fayoum 19 ، Kafr El-Sheikh 13)*
3	Two positive numbers ، one of them exceeds three times the other by 1 and the sum of their squares is 17 **What are the two numbers ?** *(El-Sharkia 04)*
4	The perimeter of a rectangle is 18 and its area is 18 cm^2 **Find its two dimensions.** *(New Valley 16)*

5	A length of a rectangle is 3 cm. more than its width and its area is 28 cm^2. **Find its perimeter.** *(El-Fayoum 12)*
6	A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm. **Find the lengths of the other two sides.** *(El-Monofia 15)*
7	A right-angled triangle in which the length of one of the sides of right-angle is 5 cm. and its perimeter is 30 cm. find the area of the triangle. (Indicating the steps of the solution) *(El-Monofia 17)*
8	The length of a rectangle is x cm. and its width is y cm. and its area = 77 cm^2. If its length decreases by 2 cm. and its width increases 2 cm. , then it will become a square. **Find the area of the square.** *(North Sinai 05)*

Solutions

A Essay Problems

1

Let the two numbers be X and y :

$\therefore X + y = 17$ (1)

, $X y = 72$ (2)

From (1) : $\therefore X = 17 - y$ (3)

Substituting from (3) in (2) :

$\therefore (17 - y)\,y = 72$ $\therefore 17y - y^2 - 72 = 0$

$\therefore y^2 - 17y + 72 = 0$ $\therefore (y - 9)(y - 8) = 0$

$\therefore y = 9$ or $y = 8$

Substituting in (3) : $\therefore X = 8$ or $X = 9$

$\therefore$ The two numbers are 8 and 9

2

Let the two numbers be X and y

$\therefore X + y = 9$ (1)

, $X^2 - y^2 = 45$ (2)

From (1) : $\therefore X = 9 - y$ (3)

Substituting from (3) in (2) : $\therefore (9 - y)^2 - y^2 = 45$

$\therefore 81 - 18y + y^2 - y^2 = 45$ $\therefore 81 - 18y = 45$

$\therefore 18y = 36$ $\therefore y = 2$

Substituting in (3) : $\therefore X = 9 - 2 = 7$

$\therefore$ The two numbers are 7 and 2

3

Let the two numbers be X and y

$\therefore X - 3y = 1$ (1)

, $X^2 + y^2 = 17$ (2)

From (1) : $\therefore X = 1 + 3y$ (3)

Substituting in (2) : $\therefore (1 + 3y)^2 + y^2 = 17$

$\therefore 1 + 6y + 9y^2 + y^2 - 17 = 0$

$\therefore 10y^2 + 6y - 16 = 0$

$\therefore 5y^2 + 3y - 8 = 0$ $\therefore (5y + 8)(y - 1) = 0$

$\therefore y = \frac{-8}{5}$ (refused) or $y = 1$

And from (3) : $\therefore X = 4$

$\therefore$ The two numbers are 1 and 4

4

Let the length of the rectangle = X cm

and the width = y cm

$\therefore (X + y) \times 2 = 18$ $\therefore X + y = 9$ (1)

, $X y = 18$ (2)

From (1) : $\therefore y = 9 - X$ (3)

Substituting in (2) : $\therefore X(9 - X) = 18$

$\therefore 9X - X^2 = 18$ $\therefore X^2 - 9X + 18 = 0$

$\therefore (X - 3)(X - 6) = 0$ $\therefore X = 3$ or $X = 6$

Substituting in (3) : $\therefore y = 6$ or $y = 3$

$\therefore$ The two dimensions are 6 cm. and 3 cm.

5

Let the length of the rectangle be X cm

and its width be y cm

$\therefore X - y = 3$ (1)

, $X y = 28$ (2)

From (1) : $\therefore X = y + 3$ (3)

Substituting from (3) in (2) :

$\therefore y(y + 3) = 28$ $\therefore y^2 + 3y - 28 = 0$

$\therefore (y + 7)(y - 4) = 0$

$\therefore y = -7$ (refused) or $y = 4$

Substituting in (3) : $\therefore X = 7$

$\therefore$ The two dimensions of the rectangle are 4 cm and 7 cm

$\therefore$ The perimeter of the rectangle $= (7 + 4) \times 2 = 22$ cm

6

Let the lengths of the two sides of the right angle be X cm. and y cm

$\therefore X + y + 13 = 30$ $\therefore X + y = 17$ (1)

, $\because X^2 + y^2 = 169$ (2)

From (1) : $\therefore X = 17 - y$ (3)

Substituting in (2) : $\therefore (17 - y)^2 + y^2 = 169$

$\therefore y^2 - 34y + 289 + y^2 - 169 = 0$

$\therefore 2y^2 - 34y + 120 = 0$ $\therefore y^2 - 17y + 60 = 0$

$\therefore (y - 12)(y - 5) = 0$ $\therefore y = 12$ or $y = 5$

Substituting in (3) : $\therefore X = 5$ or $X = 12$

$\therefore$ The side lengths of the right angle are 5 cm and 12 cm

7

Let the length of the hypotenuse = X cm

, the length of the other side = y cm

$\therefore X + y + 5 = 30$ $\therefore X + y = 25$ (1)

, $X^2 = y^2 + 25$ (2)

From (1) : $\therefore X = 25 - y$ (3)

Substituting in (2) : $\therefore (25 - y)^2 = y^2 + 25$

$\therefore 625 - 50y + y^2 - y^2 - 25 = 0$

$\therefore 600 - 50y = 0$ $\therefore 50y = 600$

$\therefore y = 12$ cm.

$\therefore$ The area of a triangle $= \frac{1}{2} \times 12 \times 5 = 30$ cm^2

8

$\because X y = 77$ (1)

, $X - 2 = y + 2$ $\therefore X = y + 4$ (2)

Substituting in (1) : $\therefore (y + 4) \times y = 77$

$\therefore y^2 + 4y - 77 = 0$ $\therefore (y + 11)(y - 7) = 0$

$\therefore y = -11$ (refused) or $y = 7$

Substituting in (2) : $\therefore X = 11$

$\therefore$ The side length of the square $= X - 2 = 9$ cm.

$\therefore$ The area of the square = 81 cm^2

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [1] : Set Of Zeroes Of A Polynomial Function

Generally

If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by z (f)

i.e. z (f) is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(X)$, z (f) :

- f denotes to the function
- $f(X)$ denotes to the rule of the function or the image of X by the function f
- z (f) denotes to the set of zeroes of the function f and it is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Remark

- If $k(X) = a$ where $a \in \mathbb{R}^*$, then $z(k) = \varnothing$
- If $k(X) = 0$, then $z(k) = \mathbb{R}$

Exercises

[A] Essay problems :-

1 Determine the set of zeroes of the polynomial functions which are defined by the following rules in $\mathbb{R}$:

$f(x) = (x-2)(x+3)+4$ *(El-Monofia 15)*

2 If the function $f : f(x) = x^3 - 2x^2 - 75$

Prove that : The number 5 is the one of the zeroes of the function f

(South Sinai 18 , Beni Suef 15)

3 If the set of zeroes of the function : $f(x) = a x^2 + x + b$ is $\{0 , 1\}$

Find the value of each two constants a and b *(Alex 17)* « $-1 , 0$ »

4 If the set of zeroes of the function f where $f(x) = a x^2 + b x + 15$ is $\{3 , 5\}$

Find the values of a and b *(El-Fayoum 19)* « $1 , -8$ »

[B] Choose the correct :

1 The set of zeroes of the function $f : f(x) = -3x$ is ……… *(Suez 18 , Giza 17)*

(a) $\{0\}$ (b) $\{-3\}$ (c) $\{-3 , 0\}$ (d) $\mathbb{R}$

2 The set of zeroes of the function $f : f(x) = 4$ is ……… *(Aswan 17)*

(a) $\{-4\}$ (b) $\{0\}$ (c) $\varnothing$ (d) $\{2\}$

3 The set of zeroes of the function $f : f(x) =$ zero is ……… *(Cairo 19 , Qena 09)*

(a) $\varnothing$ (b) $\mathbb{R} - \{0\}$ (c) $\mathbb{R}$ (d) zero

4 The set of zeroes of the function $f : f(x) = x^2 - 25$ is ……… *(Assiut 16 , South Sinai 14)*

(a) $\{5\}$ (b) $\{-5\}$ (c) $\{5 , -5\}$ (d) $\varnothing$

5	The set of zeroes of the function $f : f(X) = X^6 - 32X$ is .... *(Beni Suef 11)* (a) $\{0, 2\}$ (b) $\{2, 16\}$ (c) $\{6, 16\}$ (d) $\{0, 5\}$
6	If $f(X) = X^2 + X + 1$, then the set of zeroes of the function f is .... *(El-Fayoum 06)* (a) $\{0\}$ (b) $\{1\}$ (c) $\varnothing$ (d) $\{2\}$
7	The set of zeroes of the function $f : f(X) = X(X^2 - 2X + 1)$ is .... *(Alex. 13)* (a) $\{0, 1\}$ (b) $\{0, -1\}$ (c) $\{0\}$ (d) $\{1\}$
8	If $z(f) = \{2\}$, $f(X) = X^3 - m$, then $m =$ .... *(Qena 15, El Sharkia 14)* (a) $\sqrt[3]{2}$ (b) 2 (c) 4 (d) 8
9	If $z(f) = \{5\}$, $f(X) = X^3 - 3X^2 + a$, then $a =$ .... *(Port Said 14, Assiut 11)* (a) -50 (b) -5 (c) 5 (d) 50
10	If $\{2\}$ is the set of zeroes of the function $f : f(X) = X^2 - 2aX + a^2$, then $a =$ .... *(New Valley 14)* (a) 2 (b) -2 (c) 4 (d) -4
11	If the set of zeroes of $f : f(X) = X^2 + kX + 1$ is $\varnothing$, then k may equal .... *(El-Sharkia 15)* (a) 3 (b) 2 (c) 1 (d) -2

Solutions

A	Essay Problems
1	$f(X) = (X+2)(X-1)$ $\therefore z(f) = \{-2, 1\}$
2	$\because f(5) = (5)^3 - 2(5)^2 - 75 = 125 - 50 - 75 = 0$ $\therefore$ the number 5 is one of zeroes of the function f
3	$\because z(f) = \{0, 1\}$ $\therefore f(0) = 0$ $\therefore b = 0$ $\therefore f(X) = a X^2 + X$ $, \because f(1) = 0$ $\therefore a \times 1^2 + 1 = 0$ $\therefore a + 1 = 0$ $\therefore a = -1$
4	$\because f(3) = 0$ $\therefore 9a + 3b + 15 = 0$ $\therefore 3a + b = -5$ (1) $\because f(5) = 0$ $\therefore 25a + 5b + 15 = 0$ $\therefore 5a + b = -3$ (2) Subtracting (1) from (2) $\therefore 2a = 2$ $\therefore a = 1$ And from (1) : $\therefore b = -8$

B	Choose
1	A
2	C
3	C
4	C
5	A
6	C
7	A
8	D
9	A
10	A
11	C

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [2] : Algebraic Fractional Function

Algebraic fractional function

The algebraic fractional function is a function whose rule is in the form of an algebraic fraction whose numerator and denominator are polynomial functions

Examples for algebraic fractional functions :

- $f : f(x) = \dfrac{x-3}{x+2}$
- $g : g(x) = \dfrac{3x-1}{12x}$
- $r : r(x) = \dfrac{2x+1}{x^2+4}$
- $n : n(x) = \dfrac{3}{x-4}$
- $k : k(x) = \dfrac{2x+5}{(x-1)(x+4)}$
- $l : l(x) = \dfrac{x^2-9}{5}$

The domain of the algebraic fractional function

The domain of the algebraic fractional function is all real numbers except the numbers that make the fraction is undefined (i.e. except the set of zeroes of the denominator)

i.e. The domain of algebraic fractional function = $\mathbb{R}$ – the set of zeroes of the denominator

For example :

- The domain of $f : f(x) = \dfrac{x-3}{x+2}$ is $\mathbb{R} - \{-2\}$
- The domain of $n : n(x) = \dfrac{3}{x-4}$ is $\mathbb{R} - \{4\}$
- The domain of $g : g(x) = \dfrac{3x-1}{12x}$ is $\mathbb{R} - \{0\}$

Remember that

Dividing by zero is meaningless.

- The domain of $k : k(x) = \dfrac{2x+1}{x^2+4}$ is $\mathbb{R}$

(The denominator can not be equal to zero because there is no real value of x makes $x^2 + 4 = 0$)

- The domain of $r : r(x) = \dfrac{x^2-9}{5}$ is $\mathbb{R}$

(The denominator can not equal to zero because it is always equals 5)

Definition

If p and k are two polynomial functions ,

then the function n where $n : \mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(x) = \frac{p(x)}{k(x)}$ where : z (k) is the set of zeroes of the function k , n is called a real algebraic fractional function or briefly it is called an algebraic fraction.

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero.

i.e. The set of zeroes of the algebraic fractional function

= **the set of zeroes of the numerator – the set of zeroes of the denominator.**

For example:

- If the function n : $n(x) = \frac{x^2 + 3x}{x^2 - 9}$, then $n(x) = \frac{x(x+3)}{(x-3)(x+3)}$

i.e. $z(n) = \{0 , -3\} - \{3 , -3\} = \{0\}$

- If the function n : $n(x) = \frac{3x + 6}{x^2 + x - 2}$, then $n(x) = \frac{3(x+2)}{(x-1)(x+2)}$

i.e. $z(n) = \{-2\} - \{1 , -2\} = \varnothing$

The common domain of two algebraic fractions or more

The common domain of two algebraic fractions

is the set of real numbers that makes the two algebraic fractions identified together (at the same time)

Generally

If n_1 and n_2 are two algebraic fractions ,

and the domain of $n_1 = \mathbb{R} - X_1$ (where X_1 is the set of zeroes of the denominator of n_1)

and the domain of $n_2 = \mathbb{R} - X_2$ (where X_2 is the set of zeroes of the denominator of n_2), then :

The common domain of the two fractions n_1 and $n_2 = \mathbb{R} - \{X_1 \cup X_2\}$

$= \mathbb{R}$ – the set of zeroes of the two denominators of the two fractions.

Exercises

[A] Essay problems :-

1	**Find the common domain of the following algebraic fractions :** $\frac{3x}{x-2}$, $\frac{x+3}{x^2-9}$ *(North Sinai 09)*
2	**Find the common domain of the following algebraic fractions :** $\frac{x^2+x+1}{2x}$, $\frac{x^2-1}{x^2-x}$ *(Port Said 03)*
3	**Find the common domain of the following algebraic fractions :** $\frac{x-4}{x^2-5x+6}$, $\frac{2x}{x^3-9x}$ *(Luxor 19)*
4	**Find the common domain of the following algebraic fractions :** $\frac{x-1}{x+2}$, $\frac{x+2}{5}$, $\frac{x}{x-3}$ *(South Sinai 09)*
5	Determine the domain of the function n : $n(x) = \frac{2x+1}{x^2-5x+6}$ **, then find n (0) , n (2)** *(New Valley 08)*
6	If the domain of the function n : $n(x) = \frac{x-1}{x^2-ax+9}$ is $\mathbb{R} - \{3\}$ **, then find the value of a** *(Ismailia 19 , Souhag 18 , Beni Suef 17)* « 6 »
7	If the domain of the function f where $f(x) = \frac{x}{x^2-5x+m}$ is $\mathbb{R} - \{2 , c\}$ **, then find the value of each m and c** *(El-Sharkia 16)* « 6 , 3 »
8	If the domain of the function f where $f(x) = \frac{x+b}{x+a}$ is $\mathbb{R} - \{-2\}$ and $f(0) = 3$ **, then find the value of each a and b** *(El-Fayoum 16)* « 2 , 6 »
9	If the set of zeroes of the function f where $f(x) = \frac{ax^2-6x+8}{bx-4}$ is $\{4\}$ and its domain is $\mathbb{R} - \{2\}$ **, then find a , b** *(El-Sharkia 17)* « 1 , 2 »

[B] Choose the correct : -

1	If n_1 and n_2 are two algebraic fractions , the domain of $n_1 = \mathbb{R} - X_1$ where X_1 is the set of zeroes of the denominator of n_1 , the domain of $n_2 = \mathbb{R} - X_2$ where X_2 is the set of zeroes of the denominator of n_2 , then the common domain of n_1 and $n_2 = \mathbb{R} -$ …………… *(Port said 18)* (a) $X_1 - X_2$ (b) $X \cap X_2$ (c) $X_1 \cup X_2$ (d) $\varnothing$	C
2	The domain of the function n : $n(x) = \frac{x-2}{x^2+1}$ is ………… *(Qena 19 , Assiut 17)* (a) $\mathbb{R} - \{-1\}$ (b) $\mathbb{R} - \{1 , -1\}$ (c) $\mathbb{R} - \{1\}$ (d) $\mathbb{R}$	D
3	The domain of the algebraic fraction $\frac{x-5}{3}$ equals the domain of the algebraic fraction ………… *(El-Kalyoubia 16)* (a) $\frac{x}{x^2+1}$ (b) $\frac{x}{x-3}$ (c) $\frac{3}{x-5}$ (d) $\frac{x-5}{x-3}$	A
4	If $f(x) = \frac{x}{x-2}$, then $f(2) =$ ………… *(Qena 06)* (a) 2 (b) 1 (c) zero (d) undefined.	D
5	If the domain of the algebraic fraction n is $\mathbb{R} - \{2 , 3 , 4\}$, then n (3) = ………… *(El-Sharkia 19)* (a) 3 (b) 2 (c) 4 (d) undefined	D
6	The set of zeroes of the function $f : f(x) = \frac{2-x}{7}$ is ………… *(Cairo 16)* (a) $\{7\}$ (b) $\{2 , 7\}$ (c) $\{2\}$ (d) $\varnothing$	C
7	The set of zeroes of the function $f : f(x) = \frac{(x+1)(x-3)}{x^2-4}$ is ………… *(El-Menia 18)* (a) $\{3 , -3\}$ (b) $\{-3 , -1\}$ (c) $\{3 , -1\}$ (d) $\{2 , -2\}$	C

8	The set of zeroes of the function $f : f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is .... *(El-Gharbia 17)* (a) $\{2, -2\}$ (b) $\{-2, -1\}$ (c) $\{2, -1\}$ (d) $\{1, -1\}$	C
9	The set of zeroes of the function $f : f(x) = \frac{x^2 - 9}{x - 2}$ is .... *(Matrouh 17)* (a) $\{2\}$ (b) $\mathbb{R} - \{2\}$ (c) $\{3, -3\}$ (d) $\{3, -3, 2\}$	C
10	The common domain of the two fractions $\frac{2}{x^2 - 1}$, $\frac{5x}{x^2 - x}$ is .... *(El-Fayoum 18)* (a) $\mathbb{R} - \{1\}$ (b) $\mathbb{R} - \{0, 1\}$ (c) $\mathbb{R} - \{0, 1, -1\}$ (d) $\mathbb{R} - \{1, -1\}$	C
11	If the domain of the function $n : n(x) = \frac{x - 2}{x^2 + a}$ is $\mathbb{R}$, then a .... 0 *(El-Dakahlia 16)* (a) = (b) > (c) ≤ (d) <	B
12	If the domain of the function $n : n(x) = \frac{x + 2}{4x^2 + kx + 9}$ is $\mathbb{R} - \{\frac{-3}{2}\}$, then k = ... *(Kafr El-Sheikh 19)* (a) 15 (b) – 15 (c) 12 (d) – 12	C
13	If $x = 3$ is one of the zeroes of the function $f : f(x) = \frac{x^2 - 2x - k}{x^2 - 25}$, then k = ... *(Kafr El-Sheikh 18)* (a) 3 (b) 6 (c) – 3 (d) – 6	A
14	If $f(x) = \frac{7 + x}{7 - x}$, $x \in \mathbb{R} - \{7, -7\}$, then $f(-2)$ = ... *(El-Dakahlia 16)* (a) $\frac{-1}{f(-2)}$ (b) $\frac{-1}{f(2)}$ (c) $\frac{1}{f(2)}$ (d) $\frac{1}{f(-2)}$	C

Solutions

A	Essay Problems
1	The domain of $n_1 = \mathbb{R} - \{2\}$ $, \because n_2(x) = \frac{x+3}{(x+3)(x-3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-3, 3\}$ $\therefore$ The common domain $= \mathbb{R} - \{2, -3, 3\}$
2	The domain of $n_1 = \mathbb{R} - \{0\}$ $\because n_2(x) = \frac{x^2-1}{x(x-1)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 1\}$ $\therefore$ The common domain $= \mathbb{R} - \{0, 1\}$
3	$\because n_1(x) = \frac{(x-4)}{(x-2)(x-3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{2, 3\}$ $, \because n_2(x) = \frac{2x}{x(x+3)(x-3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, -3, 3\}$ $\therefore$ The common domain $= \mathbb{R} - \{2, 3, 0, -3\}$
4	The domain of $n_1 = \mathbb{R} - \{-2\}$ The domain of $n_2 = \mathbb{R}$ The domain of $n_3 = \mathbb{R} - \{3\}$ $\therefore$ The common domain $= \mathbb{R} - \{-2, 3\}$
5	$n(x) = \frac{2x+1}{(x-3)(x-2)}$ $\therefore$ The domain of $n = \mathbb{R} - \{3, 2\}$, $n(0) = \frac{1}{6}$ $n(2)$ is meaningless because $2 \notin$ the domain of n
6	$\because$ The domain of $n = \mathbb{R} - \{3\}$ $\therefore$ At $x = 3$, then $x^2 - a x + 9 = 0$ $\therefore 9 - 3a + 9 = 0$ $\therefore 3a = 18$ $\therefore a = 6$
7	$\because$ The domain of $f = \mathbb{R} - \{2, c\}$ $\therefore$ When $x = 2$ $\therefore x^2 - 5x + m = 0$ $\therefore 4 - 5 \times 2 + m = 0$ $\therefore m = 6$ $\therefore f(x) = \frac{x}{x^2 - 5x + 6}$ $\therefore f(x) = \frac{x}{(x-2)(x-3)}$ $\therefore$ The domain of $f = \mathbb{R} - \{2, 3\}$ $\therefore c = 3$
8	$\because$ The domain $= \mathbb{R} - \{-2\}$ $\therefore$ When $x = -2$ $\therefore x + a = 0$ $\therefore -2 + a = 0$ $\therefore a = 2$ $\therefore f(x) = \frac{x+b}{x+2}$ $, \because f(0) = 3$ $\therefore \frac{0+b}{0+2} = 3$ $\therefore \frac{b}{2} = 3$ $\therefore b = 6$
9	$\because z(f) = \{4\}$ $\therefore$ At $x = 4$ $\therefore a x^2 - 6x + 8 = 0$ $\therefore a \times 4^2 - 6 \times 4 + 8 = 0$ $\therefore 16a - 16 = 0$ $\therefore 16a = 16$ $\therefore a = 1$ $, \because$ The domain of $f = \mathbb{R} - \{2\}$ $\therefore$ At $x = 2$ $\therefore b x - 4 = 0$ $\therefore 2b - 4 = 0$ $\therefore 2b = 4$ $\therefore b = 2$

B	Choose
1	C
2	D
3	A
4	D
5	D
6	C

7	C
8	C
9	C
10	C
11	B
12	C
13	A
14	C

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [3] : Equality Of Two Algebraic Fractions

Reducing the algebraic fraction

Reducing the algebraic fraction is to put it in the simplest form.

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator.

For example :

- The following algebraic fractions are in the simplest form :

$$\frac{x-1}{x+1} , \frac{x^2}{x^2+1} , \frac{x^2+2x-1}{x^2+5}$$

because , there are no common factors between the numerator and the denominator of each of them.

- The following algebraic fractions are not in the simplest form :

$$\frac{x}{x(x+1)} , \frac{x^2+1}{x(x^2+1)} , \frac{x^2(2x-1)}{x^3}$$

because , there is a common factor between the numerator and denominator of each of them.

How to reduce the algebraic fraction

To reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly.
2. Identify the domain of the algebraic fraction before removing the common factors between the numerator and denominator.
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction.

Equality of two algebraic fractions

• If n_1 , n_2 are two algebraic fractions where : $n_1(X) = 3$, $n_2(X) = \frac{3X}{X}$

The question : is $n_2 = n_1$? The answer is : no

because : $n_1(X) = 3$ for all real values of X

but : $n_2(X) = 3$ if $X \neq 0$
, $n_2(X)$ is undefined if $X = 0$

i.e.

$n_2(X) = n_1(X)$ if $X \neq 0$

, $n_2(X) \neq n_1(X)$ if $X = 0$

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

Remark

Let n_1 and n_2 be two algebraic fractions where their domains are m_1 and m_2

If we could reduce $n_1(X)$ and $n_2(X)$ to the same fraction , it is said that n_1 and n_2 take the same values in the common domain $m_1 \cap m_2$

Exercises

[A] Essay problems :-

1	Reduce each of the following algebraic fractions to the simplest form showing the domain of each of them : $n(x) = \dfrac{x + \frac{1}{x}}{4x + \frac{4}{x}}$ *(Damietta 17)*
2	Find the common domain which makes $n_1(x) = n_2(x)$ where : $n_1(x) = \dfrac{4x^2 - 9}{6x - 9}$, $n_2(x) = \dfrac{2x^2 + 3x}{3x}$ *(Port Said 2015)*
3	Find the common domain which makes $n_1(x) = n_2(x)$ where : $n_1(x) = \dfrac{x^2 - 3x + 9}{x^3 + 27}$, $n_2(x) = \dfrac{2}{2x + 6}$ *(El-Sharkia 17)*
4	Find the common domain which makes $n_1(x) = n_2(x)$ where : $n_1(x) = \dfrac{x^2 - 4}{x^2 + x - 6}$, $n_2(x) = \dfrac{x^3 - x^2 - 6x}{x^3 - 9x}$ *(Kafr El-Sheikh 18)*
5	Find the common domain which makes $n_1(x) = n_2(x)$ where : $n_1(x) = \dfrac{x^2 + x - 12}{x^2 + 5x + 4}$, $n_2(x) = \dfrac{x^2 - 2x - 3}{x^2 + 2x + 1}$ *(Alex. 19 , Damietta 17)*

6	In each of the following , prove that $n_1 = n_2$: $n_1(X) = \frac{3X}{3X-6}$, $n_2(X) = \frac{2X}{2X-4}$ *(Souhag 06)*
7	In each of the following , prove that $n_1 = n_2$: $n_1(X) = \frac{X}{X^2-1}$, $n_2(X) = \frac{5X}{5X^2-5}$ *(Loxur 19)*
8	In each of the following , prove that $n_1 = n_2$: $n_1(X) = \frac{2X}{2X+4}$, $n_2(X) = \frac{X^2+2X}{X^2+4X+4}$ *(El-Beheira 19 , El-Menia 17)*
9	In each of the following , prove that $n_1 = n_2$: 📖 $n_1(X) = \frac{X^3-1}{X^3+X^2+X}$, $n_2(X) = \frac{(X-1)(X^2+1)}{X^3+X}$ *(Matrouh 18)*
10	In each of the following , prove that $n_1 = n_2$: $n_1(X) = \frac{X^2-X}{X^3-2X^2}$, $n_2(X) = \frac{X^2-3X+2}{X^3-4X^2+4X}$ *(El-Dakahlia 19)*
11	In each of the following , prove that $n_1 = n_2$: 📖 $n_1(X) = \frac{X^2}{X^3-X^2}$, $n_2(X) = \frac{X^3+X^2+X}{X^4-X}$ *(Souhag 19)*
12	In each of the following , show whether $n_1 = n_2$ or not (give reason) : $n_1(X) = \frac{X+5}{X^2-25}$, $n_2(X) = \frac{3}{3X-15}$ *(Assiut 18)*

13 In each of the following , show whether $n_1 = n_2$ or not (give reason) :

$n_1(X) = \frac{X^2-9}{X^2+4X+3}$, $n_2(X) = \frac{X-3}{X+1}$ *(Giza 16)*

14 In each of the following , show whether $n_1 = n_2$ or not (give reason) :

$n_1(X) = \frac{X^2-4}{X^2+X-6}$, $n_2(X) = \frac{X^2-X-6}{X^2-9}$ *(El-Gharbia 19 , Qena 18)*

15 In each of the following , show whether $n_1 = n_2$ or not (give reason) :

$n_1(X) = 1 - \frac{1}{X}$, $n_2(X) = \frac{1-X}{X}$ *(El Sharkia 19)*

[B] Choose the correct : -

1 If $n_1(X) = \frac{X^2-4}{X-2}$, $n_2(X) = X+2$, then $n_1 = n_2$ when they have the same domain which is .... *(Fayoum 03)*

(a) $\mathbb{R}$ (b) $\mathbb{R}-\{2\}$ (c) $\mathbb{R}-\{-2\}$ (d) $\mathbb{R}-\{1\}$

2 If $n_1(X) = \frac{1}{X-3}$, $n_2(X) = \frac{1}{3-X}$, then $n_1 \neq n_2$ because .... *(Souhag 04)*

(a) $n_1(X) = n_2(X)$ (b) the domain of n_1 = the domain of n_2

(c) $n_1(X) \neq n_2(X)$ (d) the domain of $n_1 \neq$ the domain of n_2

3 If $p(X) = \frac{X^2-2X}{(X+2)(X-2)}$, $q(X) = \frac{X}{X+2}$, then p = q when .... *(El-Sharkia 03)*

(a) $p(X) = q(X)$ for each $X \in \mathbb{R}-\{-2\}$

(b) $p(X) = q(X)$ in the simplest form

(c) $p(X) = q(X)$ for each $X \in \mathbb{R}-\{2, -2\}$

(d) $p(X) = q(X)$ for each $X \in \mathbb{R}$

Solutions

A	Essay Problems
1	$n(X) = \dfrac{\frac{X^2+1}{X}}{\frac{4X^2+4}{X}}$ $\therefore$ The domain of $n = \mathbb{R} - \{0\}$, $n(X) = \dfrac{X^2+1}{4X^2+4} = \dfrac{X^2+1}{4(X^2+1)} = \dfrac{1}{4}$
2	$\because n_1(X) = \dfrac{(2X-3)(2X+3)}{3(2X-3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{\frac{3}{2}\}$, $n_1(X) = \dfrac{2X+3}{3}$ $\quad \because n_2(X) = \dfrac{X(2X+3)}{3X}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0\}$, $n_2(X) = \dfrac{2X+3}{3}$ $\quad \therefore n_1(X) = n_2(X)$ For all the values of $X \in \mathbb{R} - \{\frac{3}{2}, 0\}$
3	$\because n_1(X) = \dfrac{X^2-3X+9}{(X+3)(X^2-3X+9)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-3\}$, $n_1(X) = \dfrac{1}{X+3}$, $\because n_2(X) = \dfrac{2}{2(X+3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-3\}$, $n_2(X) = \dfrac{1}{X+3}$ $\therefore n_1(X) = n_2(X)$ for all the values of $X \in \mathbb{R} - \{-3\}$
4	$\because n_1(X) = \dfrac{(X-2)(X+2)}{(X-2)(X+3)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{2, -3\}$, $n_1(X) = \dfrac{X+2}{X+3}$, $\because n_2(X) = \dfrac{X(X-3)(X+2)}{X(X-3)(X+3)}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{0, 3, -3\}$, $n_2(X) = \dfrac{X+2}{X+3}$ $\therefore n_1(X) = n_2(X)$ for all the values of $X \in \mathbb{R} - \{0, 2, 3, -3\}$
5	$\because n_1(X) = \dfrac{(X+4)(X-3)}{(X+4)(X+1)}$ $\therefore$ The domain of $n_1 = \mathbb{R} - \{-4, -1\}$, $n_1(X) = \dfrac{X-3}{X+1}$ $\quad \because n_2(X) = \dfrac{(X-3)(X+1)}{(X+1)^2}$ $\therefore$ The domain of $n_2 = \mathbb{R} - \{-1\}$, $n_2(X) = \dfrac{X-3}{X+1}$ $\quad \therefore n_1(X) = n_2(X)$ For all the values of $X \in \mathbb{R} - \{-4, -1\}$
6	$\because n_1(X) = \dfrac{3X}{3(X-2)}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{2\} \\ , n_1(X) = \dfrac{X}{X-2}\end{array}\right\}$ (1) $\because n_2(X) = \dfrac{2X}{2(X-2)}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{2\} \\ , n_2(X) = \dfrac{X}{X-2}\end{array}\right\}$ (2) From (1) and (2) : $\therefore n_1 = n_2$
7	$\left.\begin{array}{l}\because n_1(X) = \dfrac{X}{(X-1)(X+1)} \\ \therefore \text{The domain of } n_1 = \mathbb{R} - \{1, -1\}\end{array}\right\}$ (1) $\because n_2(X) = \dfrac{5X}{5(X-1)(X+1)}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{1, -1\} \\ , n_2(X) = \dfrac{X}{(X-1)(X+1)}\end{array}\right\}$ (2) From (1) and (2) : $\therefore n_1 = n_2$

8

$\because n_1(x) = \dfrac{2x}{2(x+2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-2\} \\ , n_1(x) = \dfrac{x}{x+2}\end{array}\right\}(1)$

$\because n_2(x) = \dfrac{x(x+2)}{(x+2)^2}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{-2\} \\ , n_2(x) = \dfrac{x}{x+2}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 = n_2$

9

$\because n_1(x) = \dfrac{(x-1)(x^2+x+1)}{x(x^2+x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(x) = \dfrac{x-1}{x}\end{array}\right\}(1)$

$\because n_2(x) = \dfrac{(x-1)(x^2+1)}{x(x^2+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0\} \\ , n_2(x) = \dfrac{x-1}{x}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 = n_2$

10

$\because n_1(x) = \dfrac{x(x-1)}{x^2(x-2)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0 , 2\} \\ , n_1(x) = \dfrac{x-1}{x(x-2)}\end{array}\right\}(1)$

$\because n_2(x) = \dfrac{(x-2)(x-1)}{x(x-2)^2}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0 , 2\} \\ , n_2(x) = \dfrac{x-1}{x(x-2)}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 = n_2$

11

$\because n_1(x) = \dfrac{x^2}{x^2(x-1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0 , 1\} \\ , n_1(x) = \dfrac{1}{x-1}\end{array}\right\}(1)$

$\because n_2(x) = \dfrac{x(x^2+x+1)}{x(x-1)(x^2+x+1)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{0 , 1\} \\ , n_2(x) = \dfrac{1}{x-1}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 = n_2$

12

$\because n_1(x) = \dfrac{x+5}{(x-5)(x+5)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{5 , -5\} \\ , n_1(x) = \dfrac{1}{x-5}\end{array}\right\}(1)$

$\because n_2(x) = \dfrac{3}{3(x-5)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{5\} \\ \therefore n_2(x) = \dfrac{1}{x-5}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 \neq n_2$

because the domain of $n_1 \neq$ the domain of n_2

13

$\because n_1(x) = \dfrac{(x-3)(x+3)}{(x+1)(x+3)}$

$\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{-1 , -3\} \\ , n_1(x) = \dfrac{x-3}{x+1}\end{array}\right\}(1)$

$\left.\begin{array}{l}\because n_2(x) = \dfrac{x-3}{x+1} \\ \therefore \text{The domain of } n_2 = \mathbb{R} - \{-1\}\end{array}\right\}(2)$

From (1) and (2) : $\therefore n_1 \neq n_2$

because the domain of $n_1 \neq$ the domain of n_2

14	$\because n_1(X) = \frac{(X-2)(X+2)}{(X-2)(X+3)}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{2, -3\} \\ , n_1(X) = \frac{X+2}{X+3}\end{array}\right\}$ (1) $\because n_2(X) = \frac{(X-3)(X+2)}{(X-3)(X+3)}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_2 = \mathbb{R} - \{3, -3\} \\ , n_2(X) = \frac{X+2}{X+3}\end{array}\right\}$ (2) From (1) and (2) : $\therefore n_1 \neq n_2$ because the domain of $n_1 \neq$ the domain of n_2
15	$\because n_1(X) = \frac{X}{X} - \frac{1}{X} = \frac{X-1}{X}$ $\left.\begin{array}{l}\therefore \text{The domain of } n_1 = \mathbb{R} - \{0\} \\ , n_1(X) = \frac{X-1}{X}\end{array}\right\}$ (1) $\left.\begin{array}{l}, \because \text{The domain of } n_2 = \mathbb{R} - \{0\} \\ , n_2(X) = -\frac{X-1}{X}\end{array}\right\}$ (2) From (1) and (2) : $\therefore n_1 \neq n_2$ because : $n_1(X) \neq n_2(X)$
B	Choose
1	B
2	C
3	C

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [4] : Operations On Algebraic Fractions : Part [1]

Adding and subtracting the algebraic fractions

1 Adding and subtracting two algebraic fractions having the same denominator :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{k(x)}$ and $n_2(x) = \frac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \frac{f(x)}{k(x)} + \frac{p(x)}{k(x)} = \frac{f(x) + p(x)}{k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{k(x)} - \frac{p(x)}{k(x)} = \frac{f(x) - p(x)}{k(x)}$

For example:

If $n_1(x) = \frac{x}{x-2}$ and $n_2(x) = \frac{x-1}{x-2}$, then :

- $n_1(x) + n_2(x) = \frac{x}{x-2} + \frac{x-1}{x-2} = \frac{x + x - 1}{x-2} = \frac{2x-1}{x-2}$

  where the domain of the sum is $\mathbb{R} - \{2\}$

- $n_1(x) - n_2(x) = \frac{x}{x-2} - \frac{x-1}{x-2} = \frac{x - (x-1)}{x-2} = \frac{x - x + 1}{x-2} = \frac{1}{x-2}$

  where the domain of the result is $\mathbb{R} - \{2\}$

2 Adding and subtracting two algebraic fractions having different denominators :

If $x \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$n_1(x) = \frac{f(x)}{r(x)}$ and $n_2(x) = \frac{p(x)}{k(x)}$, then :

- $n_1(x) + n_2(x) = \frac{f(x)}{r(x)} + \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) + p(x) \times r(x)}{r(x) \times k(x)}$
- $n_1(x) - n_2(x) = \frac{f(x)}{r(x)} - \frac{p(x)}{k(x)} = \frac{f(x) \times k(x) - p(x) \times r(x)}{r(x) \times k(x)}$

For example:

If $n_1(x) = \frac{5}{x-3}$ and $n_2(x) = \frac{3}{x+2}$, then :

- $n_1(x) + n_2(x) = \frac{5}{x-3} + \frac{3}{x+2} = \frac{5(x+2) + 3(x-3)}{(x-3)(x+2)} = \frac{5x + 10 + 3x - 9}{(x-3)(x+2)} = \frac{8x+1}{(x-3)(x+2)}$

where the domain of the sum is $\mathbb{R} - \{3 , -2\}$

which is the common domain of the two algebraic fractions n_1 and n_2

- $n_1(X) - n_2(X) = \frac{5}{X-3} - \frac{3}{X+2} = \frac{5(X+2) - 3(X-3)}{(X-3)(X+2)} = \frac{5X + 10 - 3X + 9}{(X-3)(X+2)} = \frac{2X + 19}{(X-3)(X+2)}$

where the domain of the result is $\mathbb{R} - \{3 , -2\}$

which is the common domain of the two algebraic fractions n_1 and n_2

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction if possible.
3. Find the common domain which will be the domain of the result.
4. Reduce each fraction separately to make the operations of addition or subtraction easier.
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible.

The properties of the operations of the addition and subtraction of the algebraic fractions :

- **Addition operation of the algebraic fractions has the following properties :**

1. Commutation.
2. Association.
3. Zero is the additive neutral (additive identity) of any algebraic fraction.
4. The additive inverse of any algebraic fraction is available.

   i.e. the additive inverse of the algebraic fraction : $\frac{g(X)}{k(X)}$ is $-\frac{g(X)}{k(X)}$, $\frac{-g(X)}{k(X)}$ or $\frac{g(X)}{-k(X)}$

For example:

The additive inverse of the algebraic fraction $\frac{2}{X-1}$

is $-\frac{2}{X-1}$ or $\frac{-2}{X-1}$ or $\frac{2}{1-X}$

Note that :

The domain of the algebraic fraction is the same domain of its additive inverse.

- **Subtraction operation of algebraic fractions has no property of the previous properties.**

Exercises

[A] Essay problems :-

1	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X}{X-4} - \frac{X+4}{X^2-16}$ *(El-Kalyoubia 18 , North Sinai 17 , Aswan 16)*
2	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2+X-6}{X+3} + \frac{X^2-4}{X+2}$ *(El-Kalyoubia 16)*
3	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2+3X}{X^2+2X-3} - \frac{X-2}{X^2-3X+2}$ *(Suez 18 , El-Dakahlia 17)*
4	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2-2X+4}{X^3+8} + \frac{X^2-1}{X^2+X-2}$ *(Damietta 19 , Assiut 08)*
5	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{2X+6}{X^2+X-6} - \frac{X^2-6X}{X^2-8X+12}$ *(El-Monofia 13)*
6	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X-6}{2X^2-15X+18} + \frac{X-5}{15-13X+2X^2}$ *(El-Dakahlia 11)*
7	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2+X-2}{X^2-1} - \frac{X+5}{X^2+6X+5}$ *(Damietta 14)*
8	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{3X+15}{X^2+7X+10} + \frac{2X^2-3X-2}{X^2-4}$ *(El-Dakahlia 15)*
9	In each of the following , find n (X) in the simplest form , showing the domain of n :

	$n(x) = \frac{3x-6}{x^2-4} - \frac{x^2-3x}{x^3-x^2-6x}$ *(Qena 12)*
10	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{x}{x-2} - \frac{x}{x+2}$ *(El-Gharbia 19)*
11	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{2}{x+3} + \frac{x+3}{x^2+3x}$ *(North Sinai 14)*
12	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{x}{x^2+2x} + \frac{x+2}{x^2-4}$ *(El-Sharkia 14 , Souhag 15)*
13	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{2x-1}{x^2-x-2} - \frac{1}{x-2}$ *(Damietta 06)*
14	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{3x-4}{x^2-5x+6} + \frac{2x+6}{x^2+x-6}$ *(Qena 17 , El-Beheira 14 , Cairo 11)*
15	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{x^2+2x+4}{x^3-8} + \frac{x^2-x-12}{x^2-9}$ *(6th October 09)*
16	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{x^2}{x-1} + \frac{x}{1-x}$ *(Giza 19 , Luxor 18)*
17	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{3x^2+6x}{x^2-4} + \frac{6}{2-x}$ *(El-Kalyoubia 05)*
18	In each of the following , find n (x) in the simplest form , showing the domain of n : $n(x) = \frac{x^2+2x+4}{x^3-8} - \frac{9-x^2}{x^2+x-6}$ *(El-Monofia 18 , Alex. 17 , El-Beheira 15)*

19	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X)=\frac{X^2+X}{X^2-1}-\frac{5-X}{X^2-6X+5}$ *(El-Dakahlia 19 , El-Menia 18 , Luxor 17)*
20	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X)=\frac{2X^2-8X}{2X^2-11X+12}+\frac{3(2X+3)}{9-4X^2}$ *(El-Sharkia 03)*
21	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X)=\frac{X+3}{X^2-9}+\frac{2X+2}{3+2X-X^2}$ *(Kafr El-Sheikh 02)*
22	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X)=\frac{3X-6}{X^2-4}-\frac{9}{2-X-X^2}$ *(El-Dakahlia 18 , El-Fayoum 12)*
23	In each of the following , find n (X) in the simplest form , showing the domain of n : 🕮 $n(X)=\frac{X-5}{2X^2-13X+15}+\frac{X+3}{15X-18-2X^2}$ *(Aswan 08)*
24	In each of the following , find n (X) in the simplest form , showing the domain of n : 🕮 $n(X)=\frac{X-3}{X^2-7X+12}-\frac{X-3}{3-X}$ *(Assiut 19 , Luxor 19)*
25	If $n(X)=\frac{X^2-5X}{X^2-8X+15}-\frac{X^2+3X+9}{X^3-27}$, then find n (X) in the simplest form and calculate the value of each of n (1) , n (5) if it is possible. *(El-Sharkia 17)*
26	Find n (X) in the simplest form , showing the domain of n where : $n(X)=\frac{X+3}{X^2+6X+9}+\frac{X+2}{X+3}$, then find n (– 3) and n (2016) if it is possible. *(El-Sharkia 16)*
27	If $n(X)=\frac{X^2-2X}{X^4-3X^3+2X^2}-\frac{4-X^2}{X^2+X-2}$, find n (X) in the simplest form , showing the domain of n , then find the S.S. of the equation : n (X) = 0 *(New Valley 13)* « ∅ »

28	**Find n (X) in the simplest form , showing the domain where :** $n(X)=\frac{X^2+X+1}{X^4-X}+\frac{X+3}{3-2X-X^2}$, and if n (a) = – 2 , find the value of a *(El-Monofia 17)* « $\frac{1}{2}$ »
29	If $f_1(X)=\frac{X-a}{X+b}$, and the set of zeroes of f_1 is $\{5\}$, and the domain of f_1 is $\mathbb{R}-\{3\}$, then find the values of a and b If $f_2(X)=\frac{X-1}{X-3}$, then find $f_1(X)+f_2(X)$ in the simplest form. *(El-Dakahlia 17)* « 5 , - 3 »
30	If the domain of the function n where $n(X)=\frac{b}{X}+\frac{9}{X+a}$ is $\mathbb{R}-\{0,4\}$, n (5) = 2 , **find the values of a and b** *(Kafr El-Sheikh 16 , El-Beheira 15 , El-Menia 14)* « – 4 , 35 »

[B] Choose the correct : -

1	If $n(X)=\frac{3}{X}+\frac{X}{3}$, then the domain of n is .... *(El-Sharkia 18)* (a) $\mathbb{R}-\{3,0\}$ (b) $\mathbb{R}-\{0\}$ (c) $\mathbb{R}-\{3\}$ (d) $\mathbb{R}$	B
2	The simplest form of $\frac{X^2+1}{X^2+4}+\frac{3}{X^2+4}$ is .... *(El-Fayoum 15)* (a) 3 (b) 4 (c) 1 (d) $\frac{1}{X^2+1}$	C
3	If $X\in\mathbb{R}-\{2\}$, then $\frac{X}{X-2}+\frac{2}{2-X}=$ .... *(Aswan 13)* (a) 1 (b) 2 (c) X (d) – 1	A
4	The additive inverse of the fraction : $\frac{X+7}{X-5}$ is .... *(El-Fayoum 12)* (a) $\frac{7-X}{X+5}$ (b) $\frac{X+7}{5-X}$ (c) $\frac{-(X+7)}{5-X}$ (d) $\frac{X-7}{5-X}$	B
5	The function f where $f(X)=\frac{X-2}{X-5}$ has an additive inverse if the domain is .... *(Kafr El-Sheikh 16)* (a) $\mathbb{R}-\{2\}$ (b) $\mathbb{R}-\{5\}$ (c) $\mathbb{R}-\{5,-2\}$ (d) $\mathbb{R}-\{2,5\}$	B
6	If $n(X)=\frac{X}{X-3}-\frac{1}{X-3}$, then the set of zeroes of the function n is .... *(Helwan 11)* (a) $\{3\}$ (b) $\{1\}$ (c) $\{-1\}$ (d) $\{-3\}$	B

Solutions

A	Essay Problems
1	$\because n(x) = \frac{x}{x-4} - \frac{x+4}{(x+4)(x-4)}$ $\therefore$ The domain of $n = \mathbb{R} - \{4 , -4\}$ $\therefore n(x) = \frac{x}{x-4} - \frac{1}{x-4} = \frac{x-1}{x-4}$
2	$\because n(x) = \frac{(x-2)(x+3)}{x+3} + \frac{(x-2)(x+2)}{x+2}$ $\therefore$ The domain of $n = \mathbb{R} - \{-3 , -2\}$ $\therefore n(x) = (x-2) + (x-2) = 2x-4$
3	$\because n(x) = \frac{x(x+3)}{(x+3)(x-1)} - \frac{x-2}{(x-2)(x-1)}$ $\therefore$ The domain of $n = \mathbb{R} - \{-3 , 1 , 2\}$ $, n(x) = \frac{x}{x-1} - \frac{1}{x-1} = \frac{x-1}{x-1} = 1$
4	$\because n(x) = \frac{x^2-2x+4}{(x+2)(x^2-2x+4)} + \frac{(x-1)(x+1)}{(x+2)(x-1)}$ $\therefore$ The domain of $n = \mathbb{R} - \{-2 , 1\}$ $\therefore n(x) = \frac{1}{x+2} + \frac{x+1}{x+2} = \frac{x+2}{x+2} = 1$
5	$\because n(x) = \frac{2(x+3)}{(x+3)(x-2)} - \frac{x(x-6)}{(x-6)(x-2)}$ $\therefore$ The domain of $n = \mathbb{R} - \{-3 , 2 , 6\}$ $\therefore n(x) = \frac{2}{x-2} - \frac{x}{x-2} = \frac{2-x}{x-2}$ $= \frac{-(x-2)}{x-2} = -1$
6	$\because n(x) = \frac{x-6}{(2x-3)(x-6)} + \frac{x-5}{(2x-3)(x-5)}$ $\therefore$ The domain of $n = \mathbb{R} - \{\frac{3}{2} , 6 , 5\}$ $\therefore n(x) = \frac{1}{2x-3} + \frac{1}{2x-3} = \frac{2}{2x-3}$
7	$\because n(x) = \frac{(x+2)(x-1)}{(x+1)(x-1)} - \frac{x+5}{(x+5)(x+1)}$ $\therefore$ The domain of $n = \mathbb{R} - \{1 , -1 , -5\}$ $\therefore n(x) = \frac{x+2}{x+1} - \frac{1}{x+1} = \frac{x+1}{x+1} = 1$
8	$\because n(x) = \frac{3(x+5)}{(x+2)(x+5)} + \frac{(2x+1)(x-2)}{(x-2)(x+2)}$ $\therefore$ The domain of $n = \mathbb{R} - \{-2 , -5 , 2\}$ $\therefore n(x) = \frac{3}{x+2} + \frac{2x+1}{x+2} = \frac{2x+4}{x+2}$ $= \frac{2(x+2)}{x+2} = 2$
9	$\because n(x) = \frac{3(x-2)}{(x-2)(x+2)} - \frac{x(x-3)}{x(x+2)(x-3)}$ $\therefore$ The domain of $n = \mathbb{R} - \{2 , -2 , 0 , 3\}$ $\therefore n(x) = \frac{3}{x+2} - \frac{1}{x+2} = \frac{2}{x+2}$
10	$\because n(x) = \frac{x}{x-2} - \frac{x}{x+2}$ $\therefore$ The domain of $n = \mathbb{R} - \{2 , -2\}$ $\therefore n(x) = \frac{x(x+2) - x(x-2)}{(x-2)(x+2)}$ $= \frac{x^2+2x-x^2+2x}{(x-2)(x+2)} = \frac{4x}{(x-2)(x+2)}$
11	$\because n(x) = \frac{2}{x+3} + \frac{x+3}{x(x+3)}$ $\therefore$ The domain of $n = \mathbb{R} - \{-3 , 0\}$ $\therefore n(x) = \frac{2x+x+3}{x(x+3)} = \frac{3x+3}{x(x+3)}$

12

$\because n(x) = \dfrac{x}{x(x+2)} + \dfrac{x+2}{(x+2)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0 , 2 , -2\}$

$\therefore n(x) = \dfrac{1}{x+2} + \dfrac{1}{x-2} = \dfrac{x-2+x+2}{(x+2)(x-2)}$

$= \dfrac{2x}{(x+2)(x-2)}$

13

$\because n(x) = \dfrac{2x-1}{(x-2)(x+1)} - \dfrac{1}{x-2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -1\}$

$\therefore n(x) = \dfrac{2x-1-x-1}{(x-2)(x+1)} = \dfrac{x-2}{(x-2)(x+1)} = \dfrac{1}{x+1}$

14

$\because n(x) = \dfrac{3x-4}{(x-3)(x-2)} + \dfrac{2(x+3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , 2 , -3\}$

$\therefore n(x) = \dfrac{3x-4}{(x-3)(x-2)} + \dfrac{2}{x-2}$

$= \dfrac{3x-4+2x-6}{(x-3)(x-2)} = \dfrac{5x-10}{(x-3)(x-2)}$

$= \dfrac{5(x-2)}{(x-3)(x-2)} = \dfrac{5}{x-3}$

15

$\because n(x) = \dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \dfrac{(x-4)(x+3)}{(x-3)(x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , 3 , -3\}$

$\therefore n(x) = \dfrac{1}{x-2} + \dfrac{x-4}{x-3} = \dfrac{x-3+(x-2)(x-4)}{(x-2)(x-3)}$

$= \dfrac{x-3+x^2-6x+8}{(x-2)(x-3)} = \dfrac{x^2-5x+5}{(x-2)(x-3)}$

16

$\because n(x) = \dfrac{x^2}{x-1} - \dfrac{x}{x-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$

$, n(x) = \dfrac{x^2-x}{x-1} = \dfrac{x(x-1)}{x-1} = x$

17

$\because n(x) = \dfrac{3x(x+2)}{(x-2)(x+2)} - \dfrac{6}{x-2}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -2\}$

$\therefore n(x) = \dfrac{3x}{x-2} - \dfrac{6}{x-2} = \dfrac{3x-6}{x-2} = \dfrac{3(x-2)}{x-2} = 3$

18

$\because n(x) = \dfrac{x^2+2x+4}{(x-2)(x^2+2x+4)} + \dfrac{(x+3)(x-3)}{(x+3)(x-2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -3\}$

$\therefore n(x) = \dfrac{1}{x-2} + \dfrac{x-3}{x-2} = \dfrac{x-2}{x-2} = 1$

19

$\because n(x) = \dfrac{x(x+1)}{(x-1)(x+1)} + \dfrac{x-5}{(x-5)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-1 , 1 , 5\}$

$\therefore n(x) = \dfrac{x}{x-1} + \dfrac{1}{x-1} = \dfrac{x+1}{x-1}$

20

$\because n(x) = \dfrac{2x(x-4)}{(2x-3)(x-4)} - \dfrac{3(2x+3)}{(2x-3)(2x+3)}$

$\therefore$ The domain of $n = \mathbb{R} - \left\{\dfrac{3}{2} , 4 , \dfrac{-3}{2}\right\}$

$\therefore n(x) = \dfrac{2x}{2x-3} - \dfrac{3}{2x-3} = \dfrac{2x-3}{2x-3} = 1$

21

$\because n(x) = \dfrac{x+3}{x^2-9} - \dfrac{2x+2}{x^2-2x-3}$

$= \dfrac{(x+3)}{(x-3)(x+3)} - \dfrac{2(x+1)}{(x-3)(x+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3 , -3 , -1\}$

$\therefore n(x) = \dfrac{1}{x-3} - \dfrac{2}{x-3} = \dfrac{-1}{x-3}$

22

$\because n(x) = \dfrac{3x-6}{x^2-4} + \dfrac{9}{x^2+x-2}$

$= \dfrac{3(x-2)}{(x-2)(x+2)} + \dfrac{9}{(x+2)(x-1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2 , -2 , 1\}$

$\therefore n(x) = \dfrac{3}{x+2} + \dfrac{9}{(x+2)(x-1)}$

$= \dfrac{3(x-1)+9}{(x+2)(x-1)} = \dfrac{3x-3+9}{(x+2)(x-1)}$

$= \dfrac{3x+6}{(x+2)(x-1)} = \dfrac{3(x+2)}{(x+2)(x-1)} = \dfrac{3}{x-1}$

23	$\because n(X)=\frac{X-5}{(X-5)(2X-3)}+\frac{X+3}{-(2X-3)(X-6)}$ $\therefore$ The domain of $n=\mathbb{R}-\left\{5,\frac{3}{2},6\right\}$ $\therefore n(X)=\frac{1}{2X-3}-\frac{X+3}{(2X-3)(X-6)}$ $=\frac{X-6-X-3}{(2X-3)(X-6)}=\frac{-9}{(2X-3)(X-6)}$
24	$\because n(X)=\frac{X-3}{(X-3)(X-4)}+\frac{X-3}{X-3}$ $\therefore$ The domain of $n=\mathbb{R}-\{3,4\}$ $\therefore n(X)=\frac{1}{X-4}+1=\frac{1}{X-4}+\frac{X-4}{X-4}=\frac{X-3}{X-4}$
25	$\because n(X)=\frac{X(X-5)}{(X-3)(X-5)}-\frac{X^2+3X+9}{(X-3)(X^2+3X+9)}$ $\therefore$ The domain of $n=\mathbb{R}-\{3,5\}$ $\therefore n(X)=\frac{X}{X-3}-\frac{1}{X-3}=\frac{X-1}{X-3}$, $n(1)=0$, $n(5)$ is undefined
26	$n(X)=\frac{X+3}{(X+3)^2}+\frac{X+2}{X+3}$ $\therefore$ The domain of $n=\mathbb{R}-\{-3\}$ $\therefore n(X)=\frac{1}{X+3}+\frac{X+2}{X+3}=\frac{X+3}{X+3}=1$, $n(-3)$ is undefined because $-3\notin$ the domain of n , $n(2016)=1$
27	$\because n(X)=\frac{X(X-2)}{X^2(X^2-3X+2)}+\frac{X^2-4}{X^2+X-2}$ $=\frac{X(X-2)}{X^2(X-2)(X-1)}+\frac{(X-2)(X+2)}{(X-1)(X+2)}$ $\therefore$ The domain of $n=\mathbb{R}-\{0,2,1,-2\}$, $n(X)=\frac{1}{X(X-1)}+\frac{X-2}{X-1}$ $=\frac{1+X^2-2X}{X(X-1)}=\frac{X^2-2X+1}{X(X-1)}$ $=\frac{(X-1)^2}{X(X-1)}=\frac{X-1}{X}$, $\because n(X)=0$ $\therefore \frac{X-1}{X}=0$ $\therefore X-1=0$ $\therefore X=1$ $\therefore$ The S.S $=\varnothing$

28	$\because n(X)=\frac{X^2+X+1}{X^4-X}-\frac{X+3}{X^2+2X-3}$ $=\frac{X^2+X+1}{X(X-1)(X^2+X+1)}-\frac{X+3}{(X+3)(X-1)}$ $\therefore$ The domain of $n=\mathbb{R}-\{0,1,-3\}$, $n(X)=\frac{1}{X(X-1)}-\frac{1}{X-1}=\frac{1-X}{X(X-1)}$ $=\frac{-(X-1)}{X(X-1)}=\frac{-1}{X}$, $\because n(a)=-2$ $\therefore \frac{-1}{a}=-2$ $\therefore -2a=-1$ $\therefore a=\frac{1}{2}$
29	$\because z(f_1)=\{5\}$ $\therefore$ at $X=5$ $\therefore X-a=0$ $\therefore 5-a=0$ $\therefore a=5$, $\because$ the domain of $f_1=\mathbb{R}-\{3\}$ $\therefore$ at $X=3$ $\therefore X+b=0$ $\therefore 3+b=0$ $\therefore b=-3$ $\therefore f_1(X)=\frac{X-5}{X-3}$ $\therefore f_1(X)+f_2(X)=\frac{X-5}{X-3}+\frac{X-1}{X-3}$ $\therefore$ The domain $=\mathbb{R}-\{3\}$, $f_1(X)+f_2(X)=\frac{X-5+X-1}{X-3}=\frac{2X-6}{X-3}=\frac{2(X-3)}{X-3}=2$
30	$\because$ The domain of $n=\mathbb{R}-\{0,4\}$ $\therefore a=-4$ $\therefore n(X)=\frac{b}{X}+\frac{9}{X-4}$, $\because n(5)=2$ $\therefore \frac{b}{5}+9=2$ $\therefore \frac{b}{5}=-7$ $\therefore b=-35$
B	**Choose**
1	**B**
2	**C**
3	**A**
4	**B**
5	**B**
6	**B**

Prep [3] – Second Term – Algebra – Unit [2] : Algebraic Fractional Functions

Lesson [4] : Operations On Algebraic Fractions : Part [2]

Multiplying and dividing the algebraic fractions

1 Multiplying the algebraic fractions

- Multiplying two algebraic fractions is similar to multiplying two fractional numbers , therefore it is better to remember together how to multiply two fractional numbers.

Remember that

$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$ (where $bd \neq 0$)

Multiplying two algebraic fractions

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$

, then : $n_1(X) \times n_2(X) = \frac{f(X)}{r(X)} \times \frac{p(X)}{k(X)} = \frac{f(X) \times p(X)}{r(X) \times k(X)}$

The steps of multiplying the algebraic fractions :

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it.
2. Factorize the numerator and the denominator of each fraction alone if it is possible.
3. Find the common domain.
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction.
5. Perform the operation of multiplication and put the result in the simplest form.

The properties of the operation of multiplying the algebraic fractions

The operation of multiplying the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity).

4 Existing the multiplicative inverses.

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(x) = \frac{p(x)}{k(x)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(x) = \frac{k(x)}{p(x)}$

and the domain of n^{-1} is $\mathbb{R}$ – the set of zeroes of each of the numerator and the denominator of any of the two fractions.

2 Dividing an algebraic fraction by another

Dividing two algebraic fractions is similar to dividing two fractional numbers, therefore it is better to remember together how to divide two fractional numbers.

Remember that

If $\frac{a}{b}$ and $\frac{c}{d}$ are two fractional numbers , $b \neq 0$ and $\frac{c}{d} \neq 0$

, then $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times$ the multiplicative inverse of the number $\frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$ (where $bd \neq 0$)

Dividing an algebraic fraction by another

If n_1 and n_2 are two algebraic fractions where : $n_1(x) = \frac{f(x)}{r(x)}$, $n_2(x) = \frac{p(x)}{k(x)}$

, then : $n_1(x) \div n_2(x) = n_1(x) \times n_2^{-1}(x) = \frac{f(x)}{r(x)} \times \frac{k(x)}{p(x)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1 and n_2^{-1}

$= \mathbb{R}$ – the set of zeroes of denominator of n_1 or denominator of n_2 or numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

Exercises

[A] Essay problems :-

1	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{3X-15}{X+3} \times \frac{4X+12}{5X-25}$ *(Luxor 13)*
2	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X+2}{X^2-4} \times \frac{2X-4}{X-3}$ *(Luxor 05)*
3	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2+2X+1}{2X-8} \times \frac{X-4}{X+1}$ *(Suez 17 , Cairo 16 , Ismailia 15)*
4	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^3-1}{X^2-2X+1} \times \frac{2X-2}{X^2+X+1}$ *(El-Dakahlia 19 , El-Kalyoubia 18 , El-Monofia 18)*
5	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{2X-10}{X^2-25} \times \frac{X^2+5X}{X-3}$ *(Qena 09)*
6	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^2-3X+4}{X^2-1} \times \frac{X^2-X}{X^2+3X}$ *(El-Kalyoubia 16 , El-Gharbia 04)*
7	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{6X^2+3X}{X+2} \times \frac{X^2+4X+4}{6X+3}$ *(Assiut 15)*
8	In each of the following , find n (X) in the simplest form , showing the domain of n : $n(X) = \frac{X^3-1}{X^2-X} \times \frac{X+3}{X^2+X+1}$ *(Alexandria 19)*

9	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{5X+5}{X+6} \times \frac{X^2+3X-18}{X^2-2X-3}$, then find n (2) if it is possible. *(Ismailia 09)*
10	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X^2+2X}{X^3-27} \times \frac{X^2+3X+9}{X+2}$, then find n (6) , n (– 2) if it is possible. *(South Sinai 17)*
11	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X^3-8}{X^2+3X-10} \times \frac{2X+6}{X^2+2X+4}$, then find $n^{-1}(X)$ when $X = 1$ *(Port Said 04)*
12	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{2X^3-16}{X^2-7X+10} \times \frac{3X^2-10X-25}{X^2+2X+4}$ *(Ismailia 09)*
13	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{3X-15}{X+3} \div \frac{5X-25}{4X+12}$ *(Luxor 18 , Beni Suef 14)*
14	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X-1}{X^2-1} \div \frac{X^2-5X}{X^2-4X-5}$ *(Matrouh 19 , El-Menia 16 , El-Beheira 15 , Aswan 14)*
15	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X^2+2X-3}{X+3} \div \frac{X^2-1}{X+1}$ *(Port Said 18 , Alexandria 13)*
16	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X^2-2X-15}{X^2-9} \div \frac{2X-10}{X^2-6X+9}$ *(El-Gharbia 18 , El-Beheira 18 , Alexandria 16)*
17	**In each of the following , find n (X) in the simplest form , showing the domain of n :** $n(X) = \frac{X^3-8}{X^2+X-6} \div \frac{X^2+2X+4}{2X+6}$ *(Alexandria 09)*

18 **In each of the following , find n (X) in the simplest form , showing the domain of n :**

$n(X) = \frac{X^2 - 2X + 1}{X^3 - 1} \div \frac{X - 1}{X^2 + X + 1}$ *(Suez 19 , El-Dakahlia 18 , El-Gharbia 17)*

19 **In each of the following , find n (X) in the simplest form , showing the domain of n :**

$n(X) = \frac{X^3 - 27}{X^2 - 9} \div \frac{X^3 + 3X^2 + 9X}{2X}$ *(El-Fayoum 09)*

20 **In each of the following , find n (X) in the simplest form , showing the domain of n :**

$n(X) = \frac{X^2 - 3X}{2X^2 - X - 6} \div \frac{2X^2 - 3X}{4X^2 - 9}$ *(Luxor 19)*

21 **In each of the following , find n (X) in the simplest form , showing the domain of n :**

$n(X) = \frac{X^2 - 9}{2X^2 + 3X} \div \frac{3X^2 + 6X - 45}{4X^2 - 9}$ *(Aswan 08)*

22 If $n(X) = \frac{X^2 - 2X}{(X - 2)(X^2 + 2)}$

First : Find $n^{-1}(X)$ and identify the domain.

Second : If $n^{-1}(X) = 3$, what is the value of X ?

(Alex. 19 , El-Kalyoubia 18 , El-Gharbia 17 , Aswan 16) « 1 »

23 If $n(X) = \frac{X^3 + 3X^2 + 2X}{X^2 + 2X}$, find $n^{-1}(X)$ in the simplest form showing the domain of n^{-1} , then find $n^{-1}(-2)$ if it is possible. *(Ismailia 08)* « undefined »

24 If $n(X) = X + \frac{X}{X - 2}$, find $n^{-1}(X)$ in the simplest form showing the domain of n^{-1}

(El-Gharbia 19)

25 If $f(X) = \frac{X^2 - 49}{X^3 - 8} \div \frac{X + 7}{X - 2}$, then find n (X) in the simplest form and identify its domain and find $f(1)$ *(Assiut 19 , El-Beheira 17 , El-Gharbia 12)* « $-\frac{6}{7}$ »

26 If $f(X) = \frac{X^2 - 2X - 15}{X^2 - 9} \div \frac{X^2 - 25}{X^2 - 3X}$ find $f(X)$ in the simplest form showing the domain of f and if $f(a) = \frac{1}{3}$ find the value of a *(Assiut 08)*

27 Find in the simplest form :

$$n(X) = \left(\frac{3X+15}{X^2+7X+10} + \frac{2X+1}{X+2} \right) \times \frac{X^3-27}{X^2+3X+9}$$

Showing the domain of n and if n (X) = 2 , find the value of X *(Suez 05) « 4 »*

[B] Choose the correct : -

1	If $n(X) = \frac{X-2}{X+5}$, then the domain of n^{-1} is .... *(Port Said 19 , Souhag 18)* (a) $\mathbb{R}$ (b) $\mathbb{R} - \{2\}$ (c) $\mathbb{R} - \{-5\}$ (d) $\mathbb{R} - \{2 , -5\}$	D
2	If $n(X) = \frac{1}{(X-2)^2}$, then the domain of n^{-1} is .... *(Cairo 18)* (a) $\mathbb{R} - \{1 , 2\}$ (b) $\mathbb{R}$ (c) $\mathbb{R} - \{2\}$ (d) $\{2\}$	C
3	If $n(X) = \frac{X}{X^2+9}$, then the domain of n^{-1} is .... *(El-Sharkia 16)* (a) $\varnothing$ (b) $\mathbb{R} - \{-3 , 3\}$ (c) $\mathbb{R}$ (d) $\mathbb{R} - \{0\}$	D
4	If $n(X) = \frac{X-2}{X^2-X-6}$, then the domain of n^{-1} is .... *(El-Beheira 17)* (a) $\mathbb{R} - \{2\}$ (b) $\mathbb{R} - \{-2 , 3\}$ (c) $\mathbb{R} - \{-2 , 2\}$ (d) $\mathbb{R} - \{-2 , 2 , 3\}$	D
6	If $n(X) = \frac{X-1}{X+2}$, then $n^{-1}(1)$ is .... *(Beni Suef 17)* (a) equal to – 1 (b) equal to zero (c) equal to 3 (d) undefined	D

Solutions

A	Essay Problems
1	$n(x)=\frac{3(x-5)}{x+3}\times\frac{4(x+3)}{5(x-5)}$ $\therefore$ The domain of $n=\mathbb{R}-\{-3,5\}$, $n(x)=\frac{12}{5}$
2	$n(x)=\frac{x+2}{(x-2)(x+2)}\times\frac{2(x-2)}{x-3}$ $\therefore$ The domain of $n=\mathbb{R}-\{2,-2,3\}$, $n(x)=\frac{2}{x-3}$
3	$n(x)=\frac{(x+1)^2}{2(x-4)}\times\frac{x-4}{x+1}$ $\therefore$ The domain of $n=\mathbb{R}-\{4,-1\}$, $n(x)=\frac{x+1}{2}$
4	$n(x)=\frac{(x-1)(x^2+x+1)}{(x-1)^2}\times\frac{2(x-1)}{(x^2+x+1)}$ $\therefore$ The domain of $n=\mathbb{R}-\{1\}$, $n(x)=2$
5	$n(x)=\frac{2(x-5)}{(x-5)(x+5)}\times\frac{x(x+5)}{x-3}$ $\therefore$ The domain of $n=\mathbb{R}-\{5,-5,3\}$, $n(x)=\frac{2x}{x-3}$
6	$n(x)=\frac{(x-4)(x+1)}{(x-1)(x+1)}\times\frac{x(x-1)}{x(x+3)}$ $\therefore$ The domain of $n=\mathbb{R}-\{0,1,-1,-3\}$, $n(x)=\frac{x-4}{x+3}$
7	$n(x)=\frac{3x(2x+1)}{x+2}\times\frac{(x+2)^2}{3(2x+1)}$ $\therefore$ The domain of $n=\mathbb{R}-\{-2,-\frac{1}{2}\}$, $n(x)=x(x+2)=x^2+2x$
8	$n(x)=\frac{(x-1)(x^2+x+1)}{x(x-1)}\times\frac{x+3}{x^2+x+1}$ $\therefore$ The domain of $n=\mathbb{R}-\{0,1\}$, $n(x)=\frac{x+3}{x}$
9	$n(x)=\frac{5(x+1)}{x+6}\times\frac{(x+6)(x-3)}{(x-3)(x+1)}$ $\therefore$ The domain of $n=\mathbb{R}-\{-6,3,-1\}$, $n(x)=5$, $n(2)=5$
10	$n(x)=\frac{x(x+2)}{(x-3)(x+3x+9)}\times\frac{x^2+3x+9}{x+2}$ $\therefore$ The domain of $n=\mathbb{R}-\{3,-2\}$, $n(x)=\frac{x}{x-3}$, $n(6)=\frac{6}{6-3}=2$, $n(-2)$ is undefined because $-2\notin$ the domain of n
11	$n(x)=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+5)}\times\frac{2(x+3)}{x^2+2x+4}$ $\therefore$ The domain of $n=\mathbb{R}-\{2,-5\}$ $\therefore n(x)=\frac{2(x+3)}{x+5}$ $\therefore n^{-1}(x)=\frac{x+5}{2(x+3)}$, the domain of $n^{-1}=\mathbb{R}-\{2,-5,-3\}$ $\therefore n^{-1}(1)=\frac{1+5}{2(1+3)}=\frac{6}{8}=\frac{3}{4}$
12	$n(x)=\frac{2(x-2)(x^2+2x+4)}{(x-2)(x-5)}\times\frac{(3x+5)(x-5)}{(x^2+2x+4)}$ $\therefore$ The domain of $n=\mathbb{R}-\{2,5\}$, $n(x)=2(3x+5)=6x+10$
13	$n(x)=\frac{3(x-5)}{x+3}\times\frac{4(x+3)}{5(x-5)}$ $\therefore$ The domain of $n=\mathbb{R}-\{-3,5\}$
14	$n(x)=\frac{x-1}{(x-1)(x+1)}\times\frac{(x-5)(x+1)}{x(x-5)}$ $\therefore$ The domain of $n=\mathbb{R}-\{1,-1,0,5\}$, $n(x)=\frac{1}{x}$

15

$n(X) = \frac{(X+3)(X-1)}{X+3} \times \frac{(X+1)}{(X-1)(X+1)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 1, -1\}$

$, n(X) = 1$

16

$n(X) = \frac{(X-5)(X+3)}{(X+3)(X-3)} \times \frac{(X-3)^2}{2(X-5)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 3, 5\}$

$, n(X) = \frac{1}{2}(X-3)$

17

$n(X) = \frac{(X-2)(X^2+2X+4)}{(X+3)(X-2)} \times \frac{2(X+3)}{X^2+2X+4}$

$\therefore$ The domain of $n = \mathbb{R} - \{-3, 2\}$ $, n(X) = 2$

18

$n(X) = \frac{(X-1)^2}{(X-1)(X^2+X+1)} \times \frac{X^2+X+1}{X-1}$

$\therefore$ The domain of $n = \mathbb{R} - \{1\}$ $, n(X) = 1$

19

$n(X) = \frac{(X-3)(X^2+3X+9)}{(X-3)(X+3)} \times \frac{2X}{X(X^2+3X+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{3, -3, 0\}$

$, n(X) = \frac{2}{X+3}$

20

$n(X) = \frac{X(X-3)}{(2X+3)(X-2)} \times \frac{(2X-3)(2X+3)}{X(2X-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-\frac{3}{2}, 2, 0, \frac{3}{2}\}$

$, n(X) = \frac{X-3}{X-2}$

21

$n(X) = \frac{(X-3)(X+3)}{X(2X+3)} \times \frac{(2X-3)(2X+3)}{3(X+5)(X-3)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, -\frac{3}{2}, -5, 3, \frac{3}{2}\}$

$, n(X) = \frac{(X+3)(2X-3)}{3X(X+5)}$

22

First : $n(X) = \frac{X(X-2)}{(X-2)(X^2+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{2\}$

$, n(X) = \frac{X}{X^2+2}$ $\therefore n^{-1}(X) = \frac{X^2+2}{X}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{2, 0\}$

Second : $\frac{X^2+2}{X} = 3$ $\therefore X^2 - 3X + 2 = 0$

$\therefore (X-2)(X-1) = 0$

$\therefore X = 2$ (refused) or $X = 1$

23

$\because n(X) = \frac{X(X+2)(X+1)}{X(X+2)}$

$\therefore$ The domain of $n = \mathbb{R} - \{0, -2\}$

$, n(X) = X+1$ $, n^{-1}(X) = \frac{1}{X+1}$

$\therefore$ The domain of $n^{-1} = \mathbb{R} - \{0, -2, -1\}$

$n^{-1}(-2)$ is undefined because $-2 \notin$ the domain of n^{-1}

24

$\because n(X) = \frac{X(X-2)+X}{X-2} = \frac{X^2-2X+X}{X-2}$

$= \frac{X^2-X}{X-2} = \frac{X(X-1)}{X-2}$

$\therefore n^{-1}(X) = \frac{X-2}{X(X-1)}$

, the domain of $n^{-1} = \mathbb{R} - \{2, 1, 0\}$

25

$n(X) = \frac{(X-7)(X+7)}{(X-2)(X^2+2X+4)} \times \frac{X-2}{X+7}$

$\therefore$ The domain of $n = \mathbb{R} - \{2, -7\}$

$, n(X) = \frac{X-7}{X^2+2X+4}$ $, n(1) = \frac{1-7}{1+2+4} = -\frac{6}{7}$

26

$f(X) = \frac{(X+3)(X-5)}{(X-3)(X+3)} \times \frac{X(X-3)}{(X-5)(X+5)}$

$\therefore$ The domain of $f = \mathbb{R} - \{3, -3, 5, -5, 0\}$

$, f(X) = \frac{X}{X+5}$ $\because f(a) = \frac{1}{3}$ $\therefore \frac{a}{a+5} = \frac{1}{3}$

$\therefore 3a = a+5$ $\therefore 2a = 5$ $\therefore a = \frac{5}{2}$

27

$n(X) = \left(\frac{3(X+5)}{(X+5)(X+2)} + \frac{2X+1}{X+2}\right) \times \frac{(X-3)(X^2+3X+9)}{(X^2+3X+9)}$

$\therefore$ The domain of $n = \mathbb{R} - \{-5, -2\}$

$, n(X) = \left(\frac{3}{X+2} + \frac{2X+1}{X+2}\right) \times (X-3)$

$= \left(\frac{2(X+2)}{X+2}\right)(X-3) = 2(X-3)$

$\because n(X) = 2$ $\therefore 2(X-3) = 2$

$\therefore X - 3 = 1$ $\therefore X = 4$

B	Choose
1	D
2	C
3	D
4	D
6	D

Prep [3] - Second Term - Algebra - Unit [3] - Probability

Lesson [1] : Operations On Events

1 The random experiment :

It is an experiment in which we can specify all its possible outcomes before performing it , but we cannot determine which outcome will occur certainly.

2 The sample space (S) :

It is the set of all possible outcomes of a random experiment.

3 The event :

It is a subset of the sample space.

4 The probability of occurrence of the event :

- It is said that an event occurred if the outcome of the random experiment is an element of this event.
- We can calculate the probability of an event (say A) from the relation :

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

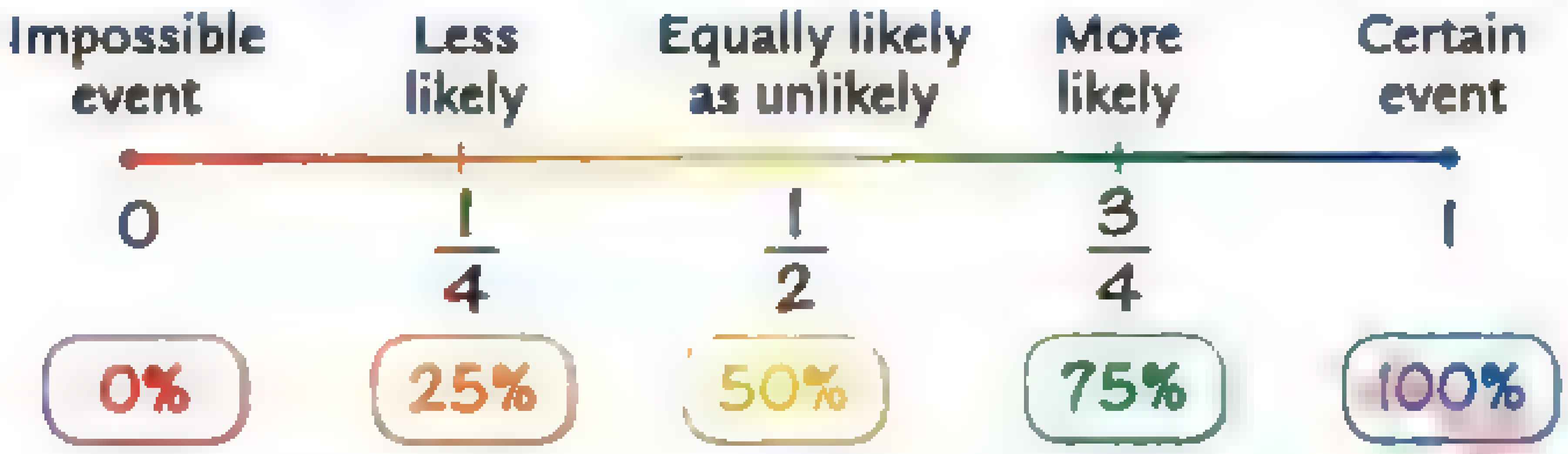
In the experiment of rolling a fair die once and observing the number appears on the upper face , if S is the sample space of the experiment and A is the event of getting an even number , then : $S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$, $A = \{2, 4, 6\}$, $n(A) = 3$

, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (i.e. The probability of occurring the event $A = \frac{1}{2}$)

Remarks

- $0 \le$ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage.

The opposite figure shows the possibility of occurring an event due to the value of its probability.



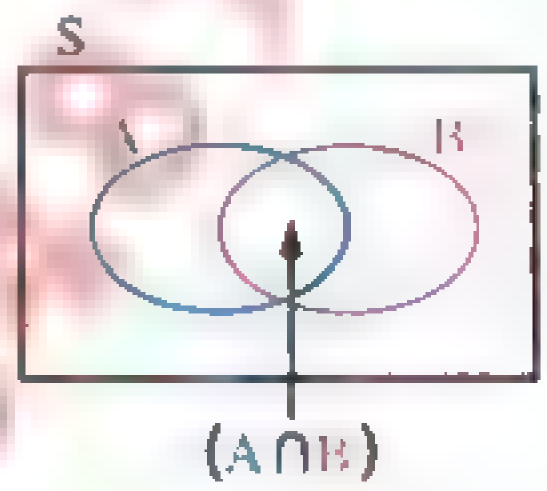
Intersection of two events

For any two events A and B of a sample space S :

The event of occurring the two events A and B together = $A \cap B$, then :

The probability of occurring the two events A and B together

$$= P(A \cap B) = \frac{n(A \cap B)}{n(S)}$$



Remarks

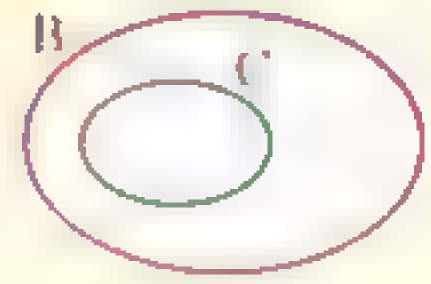
From the previous example we notice that :

1. $C \subset B$ therefore $B \cap C = C$, then we deduce that :

   The probability of occurring the two events B and C together

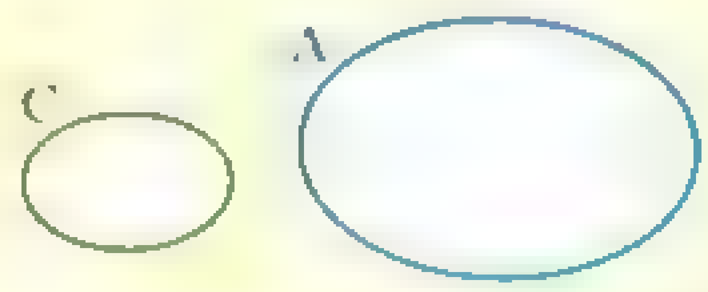
   = the probability of occurring the event C

   i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$

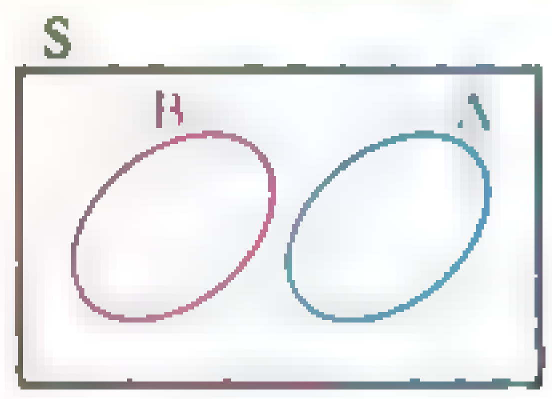


2. $A \cap C = \varnothing$ therefore it is said that the two events

   A and C are two mutually exclusive events



Mutually exclusive events



- It is said that the two events A and B are mutually exclusive if

  $A \cap B = \varnothing$, then $P(A \cap B) = 0$

i.e. The probability of their occurring together = the probability of the impossible event = 0

- It is said that some events are mutually exclusive if every pair of them is mutually exclusive.

For example: If $A \cap B = \varnothing$, $B \cap C = \varnothing$, $A \cap C = \varnothing$

, then the events A , B and C are mutually exclusive.

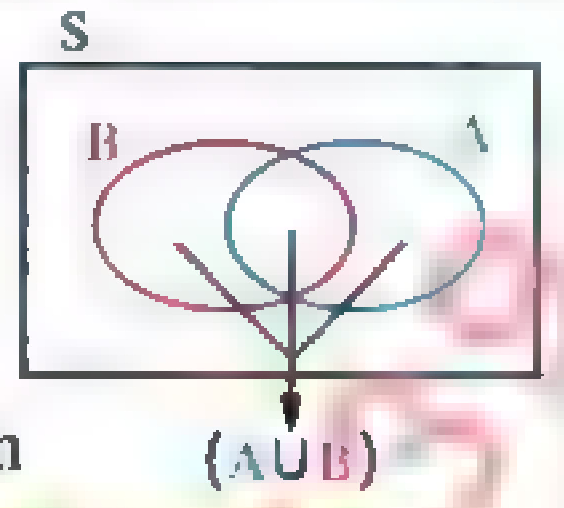
2 Union of two events

For any two events A and B from a sample space (S) :

The event of occurring the events A or the event B or both of them

(i.e. One of them at least occurs) = $A \cup B$, then :

The probability of occurring the event A or the event B or both of them



i.e. The probability of occurring one of them at least = $P(A \cup B) = \frac{n(A \cup B)}{n(S)}$

Rule

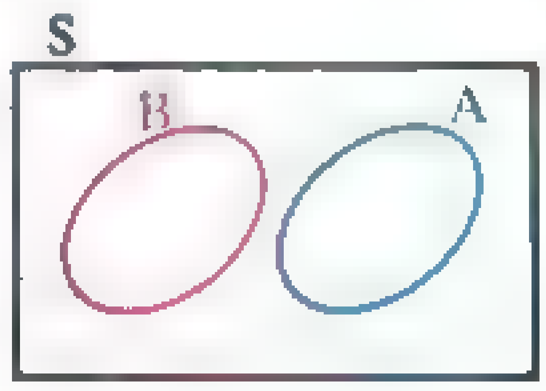
- For any two events from the sample space S of a random experiment :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

- If A and B are two mutually exclusive events , then :

$P(A \cap B) =$ zero , then :

$$P(A \cup B) = P(A) + P(B)$$



Exercises

[A] Essay problems : -

1 If A and B are two events in the sample space of a random experiment. Answer the following :

$P(A) = \frac{1}{2}$, $P(B) = \frac{2}{3}$, $P(A \cap B) = \frac{1}{3}$, then find $P(A \cup B)$ *(Port Said 13)* « $\frac{5}{6}$ »

2 If A and B are two events in the sample space of a random experiment. Answer the following :

$P(A) = \frac{3}{8}$, $P(B) = \frac{1}{2}$, $P(A \cup B) = \frac{5}{8}$, then find $P(A \cap B)$ *(Damietta 11)* « $\frac{1}{4}$ »

3 If A and B are two events in the sample space of a random experiment. Answer the following :

$P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, then find $P(A \cup B)$ in the following cases :

(i) $P(A \cap B) = \frac{1}{8}$

(ii) A and B are mutually exclusive events. *(El-Gharbia 18 , Qena 18 , Aswan 17)* « $\frac{17}{24}$, $\frac{5}{6}$ »

4 If A and B are two events from a sample space of a random experiment , $P(B) = \frac{1}{12}$ and $P(A \cup B) = \frac{1}{3}$, then find P (A) if :

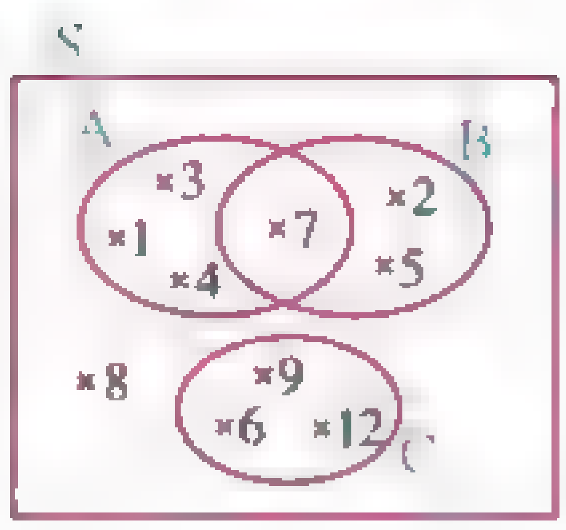
1. A and B are two mutually exclusive events.
2. $B \subset A$ *(Port Said 18 , Luxor 17 , North Sinai 14)* « $\frac{1}{4}$, $\frac{1}{3}$ »

5 If A and B are two events from the sample space of a random experiment , if $P(A) = 0.5$, $P(A \cup B) = 0.8$ and $P(B) = 2X$, then calculate the value of X if :

1. $A \subset B$
2. $P(A \cap B) = 0.1$ *(Kafr El Sheikh 16)* « 0.4 , 0.2 »

6 Use the opposite Venn diagram to find :

1. $P(A \cap B)$, $P(A \cup B)$
2. $P(A \cap C)$, $P(A \cup C)$
3. $P(B \cap C)$, $P(B \cup C)$ *(Assiut 11)*



7 S is the sample space of a random experiment where its outcomes are equal , A and B are two events from S

If the number of outcomes that leads to the occurrence of the event A equals 13 and the number of all possible outcomes of the random experiment is 24 , $P(A \cup B) = \frac{5}{6}$ and $P(B) = \frac{5}{12}$ **Find :**

1. The probability of occurrence of the event A
2. The probability of occurrence the event A and B together. *(El Menia 17 , El-Gharbia 16)* « $\frac{13}{24}$, $\frac{1}{8}$ »

8 A box contains 12 balls , 5 of them are blue , 4 are red and the left are white. A ball is randomly drawn from the box. **Find the probability that the drawn ball is :**

1. blue.
2. not red.
3. blue or red.

(Souhag 18 , Luxor 18 , Alexandria 13) « $\frac{5}{12}$, $\frac{2}{3}$, $\frac{3}{4}$ »

9 A card is randomly drawn from 20 identical cards numbered from 1 to 20
Calculate the probability that the number on the card is :

1. Divisible by 3
2. Divisible by 5
3. Divisible by 3 and divisible by 5
4. Divisible by 3 or divisible by 5

(Aswan 11) « $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{20}$, $\frac{9}{20}$ »

10 Three players A , B and C join in the competition of weight lifting. If the probability that the first player wins is equal to twice the probability of the second player to win and the probability that the player B wins is equal to the probability that the player C wins. **Find the probability that the player B or C wins , taking into consideration that one player will win.** *(Matrouh 18)* « $\frac{1}{2}$ »

[B] Choose the correct :

1 The probability of the impossible event equals .... *(Kafr El Sheikh 17 , Cairo 15)*

(a) $\varnothing$ (b) zero (c) $\frac{1}{2}$ (d) 1

2 The probability of the certain event = ... *(Qena 15)*

(a) zero (b) $\varnothing$ (c) 1 (d) – 1

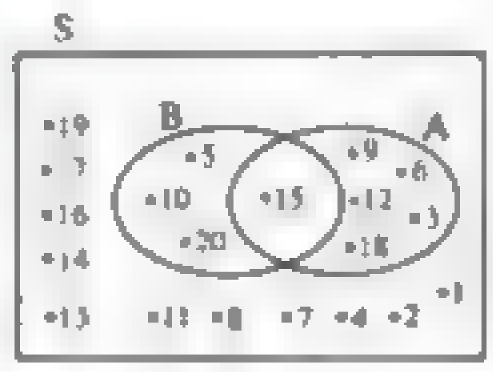
3 If A and B are two mutually exclusive events , then $P(A \cap B)$ equals ....

(a) $\varnothing$ (b) P (A) (c) P (B) (d) zero *(El-Gharbia 15)*

4	If A and B are two mutually exclusive events , then $P(A \cup B) = \cdots\cdots$ *(El Menia 16)* (a) P (A) (b) P (B) (c) $P(A \cap B)$ (d) P (A) + P (B)
5	If A and B are two events in a sample space for a random experiment , $A \subset B$, then $P(A \cap B) = \cdots\cdots$ *(Cairo 16)* (a) P (B) (b) P (A) (c) zero (d) $\varnothing$
6	If $A \subset B$, then $P(A \cup B)$ equals .... *(El Beheira 19 , El Kalyoubia 18 , Qena 17)* (a) zero (b) P (A) (c) P (B) (d) $P(A \cap B)$
7	If a regular coin is tossed once , then the probability of getting head or tail is .... *(Alexandria 14 , El-Dakahlia 13)* (a) 100 % (b) 50 % (c) 25 % (d) zero %
8	A card is drawn randomly from 20 identical cards numbered from 1 to 20 , then the probability that the number of the drawn card multiple of 7 is .... *(El Beheira 17)* (a) 10 % (b) 15 % (c) 20 % (d) 25 %
9	If a regular die is rolled once , then the probability of getting an odd number and even number together equals .... *(Alexandria 16 , El Beheira 14 , El Fayoum 12)* (a) zero (b) $\frac{1}{2}$ (c) $\frac{3}{4}$ (d) 1
10	A regular die is rolled once , if the event A is "appearing a prime number" and the event B is "appearing an odd number" , then $P(A \cap B) = \cdots\cdots$ *(El Sharkia 11)* (a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

Solutions

A	Essay Problems
1	$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{1}{2} + \frac{2}{3} - \frac{1}{3} = \frac{5}{6}$
2	$\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\therefore \frac{5}{8} = \frac{3}{8} + \frac{1}{2} - P(A \cap B)$ $\therefore P(A \cap B) = \frac{3}{8} + \frac{1}{2} - \frac{5}{8} = \frac{1}{4}$
3	[i] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{1}{2} + \frac{1}{3} - \frac{1}{8} = \frac{17}{24}$ [ii] $\because$ A and B are two mutually exclusive events $\therefore P(A \cap B) =$ zero $\therefore P(A \cup B) = P(A) + P(B) = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$
4	1 $\because$ A and B are two mutually exclusive events $\therefore P(A \cap B) =$ zero $\therefore P(A \cup B) = P(A) + P(B)$ $\therefore \frac{1}{3} = P(A) + \frac{1}{12}$ $\therefore P(A) = \frac{1}{3} - \frac{1}{12} = \frac{1}{4}$ 2 $B \subset A$ $\therefore P(A) = P(A \cup B) = \frac{1}{3}$
5	1 $\because A \subset B$ $\therefore P(B) = P(A \cup B)$ $\therefore 2x = 0.8$ $\therefore x = 0.4$ 2 $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\therefore 0.8 = 0.5 + 2x - 0.1$ $\therefore 2x = 0.8 - 0.5 + 0.1 = 0.4$ $\therefore x = 0.2$
6	1 $P(A \cap B) = \frac{1}{10}$, $P(A \cup B) = \frac{6}{10} = \frac{3}{5}$ 2 $P(A \cap C) =$ zero , $P(A \cup C) = \frac{7}{10}$ 3 $P(B \cap C) =$ zero , $P(B \cup C) = \frac{6}{10} = \frac{3}{5}$

7	1 $P(A) = \frac{13}{24}$ 2 The probability of occurrence of the two events A and B together $= P(A \cap B)$ $\because P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $\therefore \frac{5}{6} = \frac{13}{24} + \frac{5}{12} - P(A \cap B)$ $\therefore P(A \cap B) = \frac{13}{24} + \frac{5}{12} - \frac{5}{6} = \frac{1}{8}$
8	1 The probability that the drawn ball is blue $= \frac{5}{12}$ 2 The probability that the drawn ball is not red = the probability that the drawn ball is blue or white $= \frac{5}{12} + \frac{3}{12} = \frac{2}{3}$ 3 The probability that the drawn ball is blue or red $= \frac{5}{12} + \frac{4}{12} = \frac{3}{4}$
9	1 $P(A) = \frac{6}{20} = \frac{3}{10}$ 2 $P(B) = \frac{4}{20} = \frac{1}{5}$ 3 $P(A \cap B) = \frac{1}{20}$ 4 $P(A \cup B) = \frac{9}{20}$ 
10	$\because P(A) = 2\,P(B)$, $P(B) = P(C)$, $\because P(A) + P(B) + P(C) = 1$ $\therefore 2\,P(B) + P(B) + P(B) = 1$ $\therefore 4\,P(B) = 1$ $\therefore P(B) = \frac{1}{4}$ $\therefore P(C) = \frac{1}{4}$ $\because$ The event that the player B wins and the event that the player C wins are mutually exclusive $\therefore$ The probability that the player B or the player C $= P(B \cup C) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

B	Choose
1	B
2	C
3	D

4	D
5	B
6	C
7	A
8	A
9	A
10	B

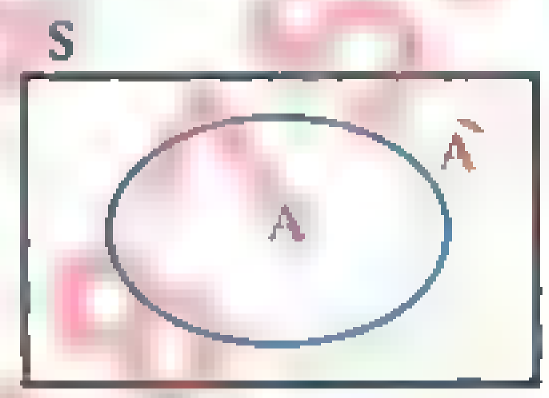
Prep [3] - Second Term - Algebra - Unit [3] - Probability

Lesson [2] : Complementary Event And The Difference Between Two Events

The complementary event

If A is an event of the sample space S ($A \subset S$) then :

the complementary event of A which is denoted by $\grave{A}$ is the event of non occurring A where $A \cup \grave{A} = S$, $A \cap \grave{A} = \varnothing$



, then the probability of non occurrence of the event A = $P(\grave{A}) = \dfrac{n(\grave{A})}{n(S)}$

Remarks !

For any event A of the sample space S it will be :

1 $A \cap \grave{A} = \varnothing$

i.e. The two events A and $\grave{A}$ are two mutually exclusive events

i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A})$ = zero

2 $A \cup \grave{A} = S$

i.e. The union of any event and the complementary event of it = the set of sample space S ,

then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

From that we deduce that :

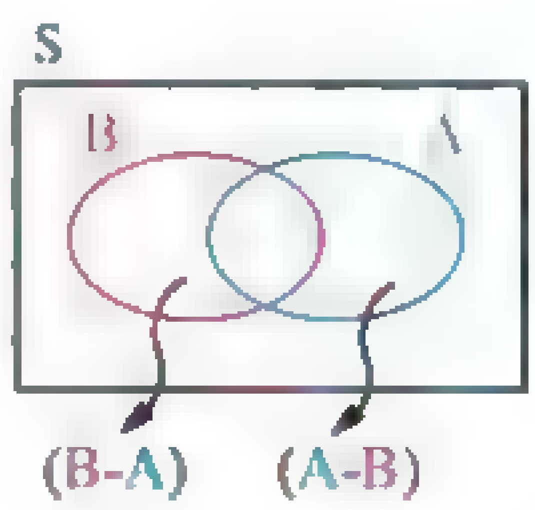
$\boxed{P(A) = 1 - P(\grave{A}) \quad , \quad P(\grave{A}) = 1 - P(A)}$

Note that :

$P(S) = \dfrac{n(S)}{n(S)} = 1$

The difference between two events

If A and B are two events of a sample space S then :



- The event of occurrence A and non occurrence B

  (**i.e.** the event of occurrence A only) = A – B

  , then the probability of occurrence the event A and non occurrence the event B = $P(A - B) = \dfrac{n(A-B)}{n(S)}$

- The event of occurrence B and non occurrence A

  (**i.e.** the event of occurrence B only) = B – A

  , then the probability of occurrence the event B and non occurrence the event A $= P(B - A) = \dfrac{n(B-A)}{n(S)}$

Remarks

If A and B are two events of a sample space (S) of a random experiment , then

- $(A - B) \cup (A \cap B) = A$

i.e. $P(A - B) + P(A \cap B) = P(A)$

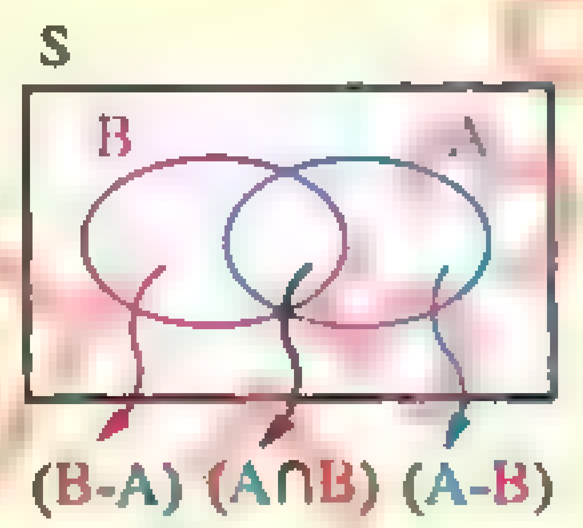
and from it : $P(A - B) = P(A) - P(A \cap B)$

Also :

- $(B - A) \cup (A \cap B) = B$

and from it : $P(B - A) = P(B) - P(A \cap B)$

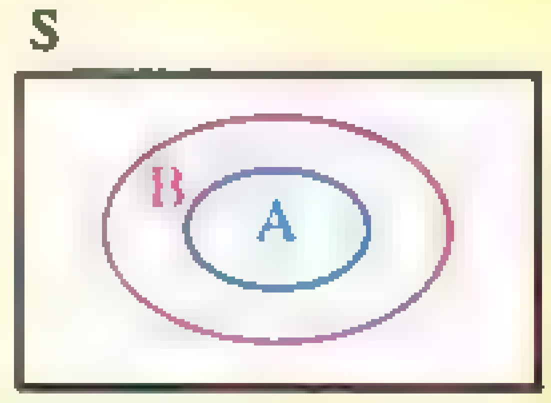
i.e. $P(B - A) + P(A \cap B) = P(B)$



Remarks

1. If A and B are two mutually exclusive events of the sample space (S) , then :
   - $A - B = A$ i.e. $P(A - B) = P(A)$
   - $B - A = B$ i.e. $P(B - A) = P(B)$

2. If A and B are two events of the sample space (S) and $A \subset B$, then :
   - $A - B = \varnothing$
   - $P(A - B) = P(\varnothing) = \frac{n(\varnothing)}{n(S)} = \text{zero}$



Remember :

1) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2) $P(A - B) = P(A) - P(A \cap B)$

3) $P(B - A) = P(B) - P(A \cap B)$

If A and B are two mutually events then :

1) $P(A \cap B) = 0$

2) $P(A \cup B) = P(A) + P(B)$

5) $P(A - B) = P(A)$

6) $P(B - A) = P(B)$

Remark [1] :

If A is $\subset$ B then :

1) $P(A \cup B) = P(B)$

2) $P(A \cap B) = P(A)$

Remark [2] :

If 1) $P(A) = 2P(A')$ then : $P(A) = \frac{2}{3}$, $P(A') = \frac{1}{3}$

If 2) $P(A) = 3P(A')$ then : $P(A) = \frac{3}{4}$, $P(A') = \frac{1}{4}$

If 3) $P(A) = 4P(A')$ then : $P(A) = \frac{4}{5}$, $P(A') = \frac{1}{5}$

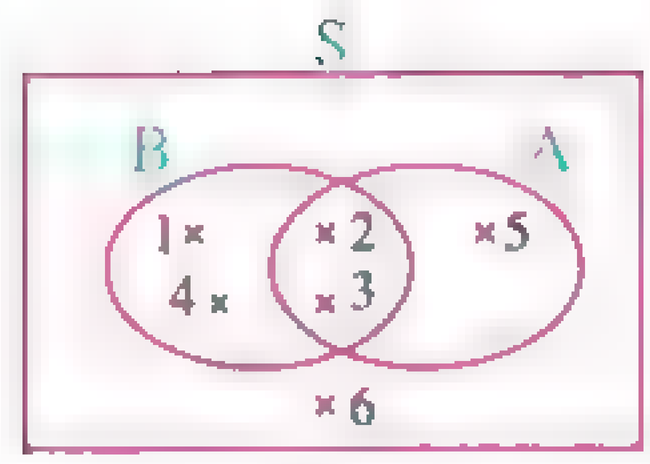
Exercises

[A] Essay problems :

1

In the opposite figure :

If A and B are two events of a sample space S of a random experiment then , **find :**



[1] $P(A \cap B)$

[2] $P(A - B)$

[3] The probability of non-occurrence of the event A

(Cairo 17) « $\frac{1}{3}$, $\frac{1}{6}$, $\frac{1}{2}$ »

2

If X and Y are two events in a sample space of a random experiment where :

$P(Y) = \frac{2}{5}$, $P(X) = P(X')$, $P(X \cap Y) = \frac{1}{5}$ **Find :**

[1] $P(X)$

[2] $P(X \cup Y)$

(Kafr El-Sheikh 18 , El-Kalyoubia 16 , El-Dakahlia 14) « $\frac{1}{2}$, $\frac{7}{10}$ »

3 If A and B are two events of a sample space of a random experiment , $P(A) = P(\grave{A})$, $P(A \cap B) = \frac{1}{16}$ and $P(B) = \frac{5}{8} P(A)$ **Find :**

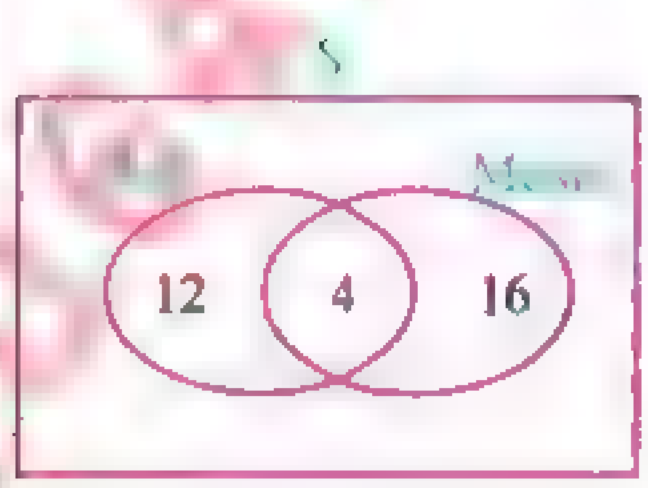
1 $P(B)$ **2** $P(A \cup B)$

3 $P(A - B)$

(El-Fayoum 19) « $\frac{5}{16}$, $\frac{3}{4}$, $\frac{7}{16}$ »

[B] Choose the correct : -

1 A class of 32 students , two sets of the students are from the lovers of art and music , their number is as in the figure. If a student is chosen randomly , then the probability that the student does not love music is ....



(a) $\frac{3}{8}$ (b) $\frac{1}{2}$ (c) $\frac{5}{8}$ (d) 1

2 If A and B are two events in a sample space the event of occurrence of A only is ....

(a) $\grave{A}$ (b) $A - B$ (c) $A \cap B$ (d) $A \cup B$

(El-Menia 15)

3 If A is an event from the sample space of the random experiment , then $P(\grave{A}) =$ ....

(El-Dakahlia 17)

(a) 1 (b) -1 (c) $1 - P(A)$ (d) $P(A) - 1$

4 If $P(A) = 4\,P(\grave{A})$, then $P(A) =$ .... *(El-Kalyoubia 18 , El-Kalyoubia 17)*

(a) 0.8 (b) 0.6 (c) 0.4 (d) 0.2

5 If A and B are two mutually exclusive events in a random experiment and $P(\grave{A}) = 0.6$, $P(A \cup B) = 0.9$, then $P(B) =$ .... *(Kafr El-Sheikh 13)*

(a) 0.5 (b) 0.4 (c) 0.6 (d) 0.3

6 If A and B are two events of the sample space of a random experiment , $P(A) = 0.6$ and $P(A \cap B) = 0.4$, then $P(A - B) =$ .... *(New Valley 14)*

(a) 0.6 (b) 0.4 (c) 0.2 (d) 0.1

7 If A and B are two events of a sample space of a random experiment , $A \subset B$, $P(A) = 0.2$ and $P(B) = 0.6$, then $P(B - A) =$ .... *(Luxor 19)*

(a) 0.6 (b) 0.2 (c) 0.8 (d) 0.4

8 For any two events C and D of a random experiment, there is : $(C - D) \cup (C \cap D) =$ .... *(El-Dakahlia 14)*

(a) 1 (b) S (c) D (d) C

Solutions

A	Essay Problems
1	[1] $P(A \cap B) = \frac{2}{6} = \frac{1}{3}$ [2] $P(A - B) = \frac{1}{6}$ [3] The probability of non occurrence of the event A $= P(\acute{A}) = \frac{3}{6} = \frac{1}{2}$
2	[1] $\because P(X) = P(\grave{X}) \quad , P(X) + P(\grave{X}) = 1$ $\therefore P(X) = \frac{1}{2}$ [2] $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$ $= \frac{1}{2} + \frac{2}{5} - \frac{1}{5} = \frac{7}{10}$
3	$\because P(A) = P(\grave{A}) , P(A) + P(\grave{A}) = 1$ $\therefore P(A) = \frac{1}{2}$ [1] $P(B) = \frac{5}{8} \times \frac{1}{2} = \frac{5}{16}$ [2] $P(A \cup B) = P(A) + P(B) - P(A \cap B)$ $= \frac{1}{2} + \frac{5}{16} - \frac{1}{16} = \frac{3}{4}$ [3] $P(A - B) = P(A) - P(A \cap B) = \frac{1}{2} \quad \frac{1}{16} = \frac{7}{16}$

B	Choose
1	A
2	B
3	C
5	A
6	A
7	C
8	D
9	D

RULES OF GEOMETRY

Solving Tow Equations Of The First Degree In Two Variables Graphically and algebraically

Prelude

- The equations $X + y = 3$, $3X - y = 7$, $y = 2X$, …
  each of them contains two variables which are X and y
  each of these two variables is of the first degree (the index of each of them is 1)
  therefore they are called equations of the first degree in two variables.

- Solving the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ means
  Finding an ordered pair from the real numbers satisfying this equation.

- Assuming an equation as $X + y = 3$
  It can be solved by making one of its two variables in an independent side as follows
  $X = 3 - y$ or $y = 3 - X$
  Then by giving one of the two variables a value and calculating the value of the other ,
  then we get the ordered pair which represents a solution of the equation

First : Solving two equations of the first degree in two variables graphically

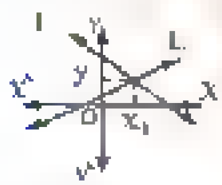
- The meaning of solving two equations graphically is finding the ordered pair or ordered pairs which satisfy the two equations simultaneously.

Since the set of solution of the equation of the first degree in two variables in $\mathbb{R} \times \mathbb{R}$ is represented graphically by a straight line

Then to solve the two equations graphically, we do as follows :

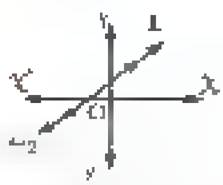
In the Cartesian plane draw the two straight lines which represent the two equations to be L_1 and L_2 , then the S.S. is the point of intersection of the two straight lines L_1 and L_2 , then we have three cases.

1 L_1 and L_2 intersect at the point (X_1 , y_1)



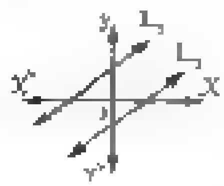
- There is a unique solution (X_1 , y_1)
- The S.S. = $\{(X_1 , y_1)\}$

2 L_1 and L_2 are coincident



- There is an infinite number of solutions

3 L_1 and L_2 are parallel



- There is no solution
- The S.S. = $\varnothing$

Notice that :

Each point belongs to this straight line determines a solution of the equation.

The equation of the first degree in two variables has an infinite number of solutions in $\mathbb{R} \times \mathbb{R}$

Remark

We can recognise the number of solutions of any two equations of the first degree in two variables by knowing the slope of the straight line and determining the point of its intersection with y-axis as follows :

We find the slopes of the two straight lines

If

- $m_1 = m_2$: We find the points of intersection of the two straight lines with y-axis
  - If
    - The two points are equals: Then the two straight lines are coincident and the number of solutions is an infinite number
    - The two points are different: Then the two straight lines are parallel and the number of solutions = 0
- $m_1 \neq m_2$: The two straight lines intersect at one point, then we say the number of solutions = 1

Second Solving two equations of the first degree in two variables algebraically

This method depends on removing one of the two variables to get an equation of the first degree in one variable , then we get the value of this variable by solving this equation

Then we substitute by this value in any of the two equations to get the value of the other variable which we have removed before

For that purpose , we follow one of the two methods

1 Substituting method. **2** Omitting method.

In the following , we will explain each of the two methods

1 Substituting method

The following example shows how to use the substituting method to solve two equations of the first degree in two variables algebraically.

Set of zeroes Of Polynomial Function

Generally

If f is a polynomial function in X , then the set of values of X which makes $f(X) = 0$ is called the set of zeroes of the function f and is denoted by $z(f)$

i.e. $z(f)$ is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Notice the difference among f , $f(X)$, $z(f)$:

- f denotes to the function
- $f(X)$ denotes to the rule of the function or the image of X by the function f
- $z(f)$ denotes to the set of zeroes of the function f and it is the solution set of the equation $f(X) = 0$ in $\mathbb{R}$

Algebraic fractional Function

Definition

If p and k are two polynomial functions , $z(k)$ is the set of zeroes of the function k ,

then the function n where $n : \mathbb{R} - z(k) \longrightarrow \mathbb{R}$, $n(X) = \dfrac{p(X)}{k(X)}$

n is called a real algebraic fractional function or briefly it is called an algebraic fraction

Remark

The set of zeroes of the algebraic fractional function is the set of values which makes its numerator equals zero and its denominator does not equal zero

i.e. The set of zeroes of the algebraic fractional function

= the set of zeroes of the numerator − the set of zeroes of the denominator

For example

- If the function n : $n(X) = \dfrac{X^2 + 3X}{X^2 - 9}$, then $n(X) = \dfrac{X(X+3)}{(X-3)(X+3)}$

i.e. $z(n) = \{0 , -3\} - \{3 , -3\} = \{0\}$

- If the function n : $n(X) = \dfrac{3X + 6}{X^2 + X - 2}$, then $n(X) = \dfrac{3(X+2)}{(X-1)(X+2)}$

i.e. $z(n) = \{-2\} - \{1 , -2\} = \varnothing$

- The common domain of two algebraic fractions is the set of real numbers that makes the two algebraic fractions identified together (at the same time)

- **Assume that we have the two algebraic fractions n_1 and n_2 where :**

$n_1(X) = \frac{3}{X - 2}$ and $n_2(X) = \frac{5X}{X^2}$,

then the domain of n_1 (say) $m_1 = \mathbb{R} - \{2\}$ (because n_1 is undefined when $X = 2$)

and the domain of n_2 (say) $m_2 = \mathbb{R} - \{1 , -1\}$ (because n_2 is undefined when $X = 1$ or $X = -1$)

According to that ,

$= \mathbb{R}$ – the set of zeroes of the two denominators

(because n_1 and n_2 are undefined together when $X = 2$ or $X = 1$ or $X = -1$)

Equality Of two algebraic Functions

Reducing the algebraic fraction

Definition

It is said that the algebraic fraction is in its simplest form if there are no common factors between its numerator and denominator

From the previous , to reduce the algebraic fraction , we do as follows :

1. Factorize each of the numerator and denominator perfectly
2. Identify the domain of the algebraic fraction before removing the common *factors between the numerator and denominator*
3. Remove the common factors between the numerator and denominator to get the simplest form of the algebraic fraction

Equality of two algebraic fractions

It is said that the two algebraic fractions n_1 and n_2 are equal (i.e. $n_1 = n_2$) if the two following conditions are satisfied together :

1. The domain of n_1 = the domain of n_2
2. $n_1(X) = n_2(X)$ for each $X \in$ the common domain.

Operations On the algebraic fractions

First : Adding and subtracting the algebraic fractions

1 Adding and subtracting two algebraic fractions having the same denominator

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$$n_1(X) = \frac{f(X)}{k(X)} \text{ and } n_2(X) = \frac{p(X)}{k(X)} \text{ , then :}$$

- $n_1(X) + n_2(X) = \frac{f(X)}{k(X)} + \frac{p(X)}{k(X)} = \frac{f(X) + p(X)}{k(X)}$
- $n_1(X) - n_2(X) = \frac{f(X)}{k(X)} - \frac{p(X)}{k(X)} = \frac{f(X) - p(X)}{k(X)}$

2 Adding and subtracting two algebraic fractions having different denominators

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where

$$n_1(X) = \frac{f(X)}{r(X)} \text{ and } n_2(X) = \frac{p(X)}{k(X)} \text{ , then :}$$

- $n_1(X) + n_2(X) = \frac{f(X)}{r(X)} + \frac{p(X)}{k(X)} = \frac{f(X) \times k(X) + p(X) \times r(X)}{r(X) \times k(X)}$
- $n_1(X) - n_2(X) = \frac{f(X)}{r(X)} - \frac{p(X)}{k(X)} = \frac{f(X) \times k(X) - p(X) \times r(X)}{r(X) \times k(X)}$

The steps of adding or subtracting two algebraic fractions :

1. Arrange the terms of each of the numerator and denominator of each fraction descendingly or ascendingly according to the powers of any variable in it.
2. Factorize the numerator and the denominator of each fraction if possible
3. Find the common domain which will be the domain of the result
4. Reduce each fraction separately to make the operations of addition or subtraction easier
5. Unify the denominators.
6. Perform the operations of addition or subtraction of the terms of the numerators.
7. Put the final result in the simplest form if possible

The properties of the operations of the addition and subtraction of the algebraic fractions

- The addition operation of the algebraic fractions has the following properties

1. Commutation
2. Association
3. Zero is the additive neutral (additive identity) of any algebraic fraction
4. The additive inverse of any algebraic fraction is available

i.e. the additive inverse of the algebraic fraction $\frac{g(X)}{k(X)}$ is $\frac{-g(X)}{k(X)}$, $-\frac{g(X)}{k(X)}$ or $\frac{g(X)}{-k(X)}$

The Operations On the algebraic fractions

Second : Multiplying and Dividing the algebraic fractions

(1) Multiplying the algebraic fractions

Remark

Notice the reduction of the numerator of the first number with the denominator of the second number and the numerator of the second number with the denominator of the first number.

- The following shows how to multiply two algebraic fractions .

Multiplying two algebraic fractions

If $X \in$ the common domain of the two algebraic fractions n_1 and n_2 where :

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$

, then $n_1(X) \times n_2(X) = \frac{f(X)}{r(X)} \times \frac{p(X)}{k(X)} = \frac{f(X) \times p(X)}{r(X) \times k(X)}$

For example :

If $n_1(X) = \frac{2}{X}$, $n_2(X) = \frac{X}{X-1}$,

then $n_1(X) \times n_2(X) = \frac{2}{X} \times \frac{X}{X-1} = \frac{2 \times X}{X(X-1)}$

where the domain of the product $= \mathbb{R} - \{0 , 1\}$

, $n_1(X) \times n_2(X) = \frac{2}{X-1}$

Notice that

The domain of the product is the common domain of the two algebraic fractions before reduction

The steps of multiplying the algebraic fractions

1. Arrange the terms of each of the numerator and the denominator of each fraction alone descendingly or ascendingly according to the powers of any symbol in it
2. Factorize the numerator and the denominator of each fraction alone if it is possible
3. Find the common domain
4. Remove the common factors between the numerator and the denominator of each fraction and between the numerator of a fraction and the denominator of another fraction
5. Perform the operation of multiplication and put the result in the simplest form

The properties of the operation of multiplying the algebraic fractions :

The operation of multiplying the algebraic fractions has the following properties :

1. Commutation.
2. Association.
3. One is the multiplicative neutral (the multiplicative identity)
4. Existing the multiplicative inverses

The multiplicative inverse of the algebraic fraction :

If n is an algebraic fraction where $n(X) = \frac{p(X)}{k(X)} \neq 0$

, then n has a multiplicative inverse which is the algebraic fraction n^{-1} where $n^{-1}(X) = \frac{k(X)}{p(X)}$

and the domain of n^{-1} is $\mathbb{R}$ − the set of zeroes of each of the numerator and the denominator of any of the two fractions.

For example:

If $n(X) = \frac{X+1}{X-5}$, then $n^{-1}(X) = \frac{X-5}{X+1}$

where the domain of $n = \mathbb{R} - \{5\}$

and the domain of $n^{-1} = \mathbb{R} - \{5, -1\}$

Note that

$n(X)$ and $n^{-1}(X)$ each of them is the reciprocal of the other

i.e the numerator of each of them is a denominator for the other

(1) Dividing an algebraic fractions by another

Dividing an algebraic fraction by another .

If n_1 and n_2 are two algebraic fractions where

$n_1(X) = \frac{f(X)}{r(X)}$, $n_2(X) = \frac{p(X)}{k(X)}$, then $n_1(X) \div n_2(X) = n_1(X) \times n_2^{-1}(X) = \frac{f(X)}{r(X)} \times \frac{k(X)}{p(X)}$

where the domain of $n_1 \div n_2$ = the common domain of each of n_1, n_2 and n_2^{-1}

$= \mathbb{R} -$ the set of zeroes of the denominator of n_1 or the denominator of n_2

or the numerator of n_2

$= \mathbb{R} - \{z(r) \cup z(p) \cup z(k)\}$

THE PROBABILITY

- We can calculate the probability of an event (say A) from the relation

$$P(A) = \frac{\text{The number of elements of the event A}}{\text{The number of elements of the sample spaces}} = \frac{n(A)}{n(S)}$$

For example:

In the experiment of rolling a fair die once and observing the number appears on the upper face, if S is the sample space of the experiment and A is the event of getting an even number, then

$S = \{1, 2, 3, 4, 5, 6\}$, $n(S) = 6$, $A = \{2, 4, 6\}$, $n(A) = 3$

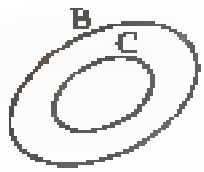
, then $P(A) = \frac{n(A)}{n(S)} = \frac{3}{6} = \frac{1}{2}$ (i.e. The probability of occurring the event $A = \frac{1}{2}$)

Remarks

- Zero ≤ the probability of any event ≤ 1
- Probability can be written as a fraction or percentage

Remarks

From the previous example we notice that :



1 $C \subset B$ therefore $B \cap C = C$, then we deduce that

The probability of occurring the two events B and C together

= the probability of occurring the event C

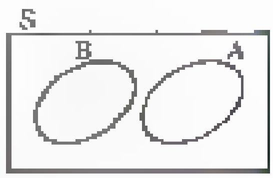
i.e. $P(B \cap C) = P(C) = \frac{n(C)}{n(S)}$

2 $A \cap C = \varnothing$ therefore it is said that the two events A and C are two mutually exclusive events

, then we can deduce that

The probability of occurring the event A or C $= P(A \cup C) = P(A) = \frac{n(A)}{n(S)}$

Mutually exclusive events

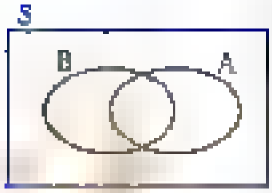


- It is said that the two events A and B are mutually exclusive if

$A \cap B = \varnothing$, then $P(A \cap B) = 0$

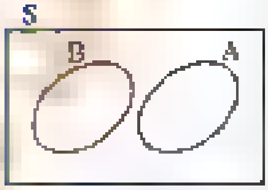
i.e. The probability of their occurring together = the probability of the impossible event = 0

Rule :



- For any two events from the sample space S of a random experiment

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



- If A and B are two mutually exclusive events , then

$P(A \cap B) = \text{zero}$, then

$$P(A \cup B) = P(A) + P(B)$$

Remarks

For any event A of the sample space S it will be :

1 $A \cap \grave{A} = \varnothing$

i.e. The two events A and $\grave{A}$ are two mutually exclusive events

i.e. Occurring one of them prevents the occurring of the other , then $P(A \cap \grave{A}) = \text{zero}$

2 $A \cup \grave{A} = S$

i.e. The union of any event and the complementary event of it is the set of sample space S ,

then $P(A \cup \grave{A}) = P(A) + P(\grave{A}) = P(S) = 1$

From that we deduce that :

$$P(A) = 1 - P(\grave{A}) \ , \ P(\grave{A}) = 1 - P(A)$$

Note that :

$$P(S) = \frac{n(S)}{n(S)} = 1$$

Remarks

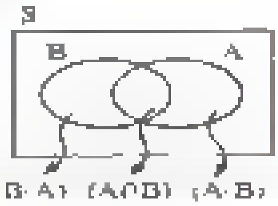
If A and B are two events of a sample space (S) of a random experiment ,

then $(A - B) \cup (A \cap B) = A$

i.e. $P(A - B) + P(A \cap B) = P(A)$

Also : $(B - A) \cup (A \cap B) = B$

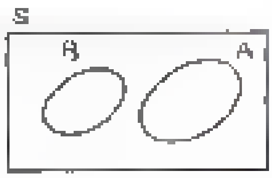
i.e. $P(B - A) + P(A \cap B) = P(B)$



Remarks

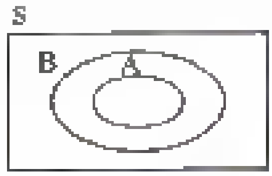
1 If A and B are two mutually exclusive of the sample space (S) , then

- $A - B = A$ *i.e.* $P(A - B) = P(A)$
- $B - A = B$ *i.e.* $P(B - A) = P(B)$



2 If A and B are two events of the sample space (S) and $A \subset B$, then

- $A - B = \varnothing$
- $P(A - B) = P(\varnothing) = \frac{n(\varnothing)}{n(S)} = \text{zero}$



Questions Part (1)

Algebra and Statistics

General Exercise on Equations

(1) Solving two equations of first degree of two variables algebraically and graphically:

First: Complete the following:

1) The two equations $x = 4$, $y - 3 = 0$ represent two straight lines intersect at the point ....

2) The two equations $x = -1$, $y + 1 = 0$ represent two straight lines intersect at a point lies on .... quadrant

3) The solution set of two equations $x + 1 = 0$, $y + 2 = 0$ is ....

4) The solution set of the two equations $x + y = 0$, $y - 5 = 0$ is........

5) The solution set of the two equations $x + 3y = 4$, $3y + x = 1$ is ....

6) The solution set of the two equations $4x + y = 6$, $8x + 2y = 12$ is ....

7) If the two equations : $x + 3y = 4$, $x + ay = 7$ represent two parallel straight lines, then a = ...

8) If the two equations : $x + 2y = 1$, $2x + ky = 2$ has one and one solution then k ≠ ......

Second : Choose:

1) The point of intersection of the two straight lines $y = 2$ and $x + y = 6$ is ....

a) (2 , 6) b) (2 , 4) c) (4 , 2) d) (6 , 2)

2) The point of intersection of the two straight lines $2x - y = 3$ and $2x + y = 5$ lies on the .... quadrant.

a) first b) second c) third d) fourth

3) If the point of intersection of the two straight lines $x = 1$ and $y = 5a$ lies on the fourth quadrant, then a may equal ....

a) -5 b) zero c) 1 d) 5

4) The two straight lines $x + 5y - 1$, $x + 5y - 8 = 0$ are ....

a) parallel b) coincide

c) intersect and non perpendicular d) perpendicular

5) The two straight lines $3x + 4y = 1$, $6x + 8y = 2$ are ..

a) parallel b) coincide

c) intersect and non perpendicular d) perpendicular

6) The two straight lines $3x = 7$, $2y = 9$ are .....

a) parallel b) coincide

c) intersect and non perpendicular d) perpendicular

7) The two straight lines $x - 1 = 0$, $x + y = 5$ are ....

a) parallel b) coincide

c) intersect and non perpendicular d) perpendicular

8) The solution set of the two equations $x + y = 0$ and $y - 1 = 0$ is ....

a) (-1 , 1) b) – 1 , 1 c) {-1 , 1} d) {(-1 , 1)}

9) The solution set of the two equations $x + 1 = 0$ and $y - 2 = 0$ is ....

a) {(1 , 2)} b) {(1 , -2)} c) {(-1 , 2)} d) {(-1 , -2)}

10) The number of solutions of the two equations $x + y = 2$ and $x + y = 0$ is ....

a) zero b) one

c) two d) infinite numbers

11) The number of solutions of the two equations $x + y = 2$ and $x + y - 3 = 0$ is ....

a) zero b) one

c) two d) infinite numbers

12) If the two equations $x + 4y = 7$ and $3x + ky = 21$ has infinite numbers of solution then k = ...

a) 4 b) 7 c) 12 d) 21

Third : find the solution set for each pair of the following equations graphically:

1) $x = 1$ $\frac{1}{3}y = = -1$

2) $\frac{1}{2}x = 2$, $\frac{6}{y} = 3$

3) $y = 3$ $2x + y = 7$

4) $x - 2 = 0$, $x + y = 5$

5) $y = x + 5$, $y = x$

6) $y + x = 7$, $y = 2x + 1$

7) $2x + y = 1$, $x + 2y = 5$

8) $3x - y + 9 = 0$, $y - 2x - 7 = 0$

9) $3x - 2y - 14 = 0$ $2x + 3y + 8 = 0$

10) $2y = 8y + 7$, $4x - 6y - 14 = 0$

Fourth : Find the solution set for each pair of the following equations graphically:

1) $y = 3$, $y = 2x - 4$

2) $x = 2$, $y = 3x + 1$

3) $y = x + 1$ $y = 2x - 1$

4) $x + y = 4$, $2x - y = 2$

5) $x + 5y = 4$, $2x - 5y = 11$

6) $y = 3x + 4$, $y = 2x + 3$

7) $3x + 4y = 7$, $2x - y = 1$

8) $y = \frac{1}{2}x$, $y + x = 9$

9) $2x + y = 5$, $x - 2y = 5$

10) $\frac{x}{2} + \frac{3y}{2} = 1$, $\frac{x}{4} + \frac{y}{3} = \frac{1}{2}$

Fifth : Find the solution set for each pair of the following equations graphically and algebraically :

1) $y = 2x + 7$, $x + 2y = 4$

2) $3x - y + 4 = 2$, $y = 2x + 3$

3) $y = x + 4$, $x + y = 4$

4) $x - y = 4$, $3x + 2y = 7$

5) $2x + y = 1$, $x + 2y = 5$

Sixth: Answer the following questions:-

1) The sum of two rational numbers is 63, and the difference between them is 12, find the two numbers

2) If three times a number is added to twice a second number the sum is 19, and if the first number is added to three times the second number the sum is 16, find the two numbers

3) The sum of two rational numbers is 12, and three times the smallest number exceeds than twice the greatest number by one find the two numbers

4) A rational number in the simplest form, if 3 is subtracted from both numerator and denominator it became $\frac{5}{6}$ and if 5 is added to both numberator and denominator it became $\frac{13}{14}$ find this number.

5) Find the number which formed from two digits if their sum is 11 and twice the units digit exceeds than three times the tens digit by 2

6) Find the number which formed from two digit, if their sum is 5 and if the two digits are exchanged then the resulting number decreases than the original number by 9

7) Since 6 years ago the age of a man was six times his son's age after ten years the age of this man will be double his son's age Find the age of both of them.

8) The length of a rectangle exceeds 3 cm. than its width, if twice the length decrease 2 cm, than four times its width. Find the length and the width of the rectangle.

9) A rectangle of perimeter 32cm. if its length decreases 1cm. and its width increases 3cm, it will be a square. Find the area of the square.

10) Two complementary angles, if the measure of one of them is 30° more than the measure of the other, find the measure of each of them

Exercises on solving second degree equation:

First : Choose the correct answer from the given ones:

1) The curve of the function f such that $f(x) = x^2 - 3x + 2$ cuts x-axis at the two points ......

a) (2, 0) , (3 , 0) b) (2 , 0) , (1 , 0)

c) (-2 , 0) , (-1 , 0) d) (2, 0) , (-1 , 0)

2) The solution set of the equation $2x^2 + 5x = 0$ is ......

a) {0 , 5} b) $\{0 , \frac{-5}{2}\}$ c) {2 , 5} d) Ø

3) The solution set of the equation $x^2 - 4x + 4 = 0$ is ......

a) {(-2 , 2)} b) {(4, 1)} c) {2} d) φ

4) The solution set of the equation $x^2 + 5 = 0$ is .

a) $\{\sqrt{5}, -\sqrt{5}\}$ b) $\{-\sqrt{5}\}$ c) $\{\sqrt{5}\}$ d) φ

5) In the equation : $ax^2 + bx + c = 0$, if $b^2 - 4ac > 0$, then the number of roots equals

a) 1 b) 2 c) 0 d) undetermined

Second: find the solution set for each pair of the following equations by using the formula:

1) $x^2 - 2x - 4 = 0$ knowing that $\sqrt{5} \simeq 2.24$

2) $x^2 = 2(x + 6)$ knowing that $\sqrt{52} \simeq 7.2$

3) $(x - 1)^2 = 10$ knowing that $\sqrt{10} \simeq 3.16$

4) $x^2 - 2(x + 3) = 0$ knowing that $\sqrt{7} \simeq 2.65$

5) $(x - 3)^2 - 3(x - 3) + 1 = 0$ knowing that $\sqrt{5} \simeq 2.24$

6) $1 - \frac{2}{x} = \frac{2}{x^2}$ (where $x \neq 0$) knowing that $\sqrt{3} \simeq 1.73$

7) $9x^2 - 24x + 16 = 0$

8) $x^2 = 2(x - 6)$

9) $x + \frac{4}{x} + 1 = 0$ (where $x \neq 0$)

10) If $x^4 + 2x^2 - 1 = 0$

Then use the formula to prove that $x^2 = \sqrt{2} - 1$

Third : Answer the following questions:

1) Graph the function f where $f(x) = x^2 - 3x + 2$, $x \in [-1, 4]$, then from the graph find

(a) The vertex point of the curve

(b) The maximum or minimum value of the function f

(c) The solution set of the equation $x^2 - 3x + 2 = 0$

2) Graph the function f where $f(x) = x^2 - 4x - 2$, $x \in [-1, 5]$, then from the graph find

(a) The maximum or minimum value of the function f

(b) The solution set of the equation $f(x) = 0$

3) Graph the function f where $f(x) = 3 - 2x - x^2$, $x \in [-4, 2]$, then from the graph find

(a) The vertex point of the curve

(b) The two roots of the equation $x^2 + 2x - 3 = 0$

4) Graph the function f where $f(x) = x^2 + 2x + 3$, $x \in [-3, 1]$, then from the graph find

(a) The vertex point of the curve.

(b) The minimum value of the function f.

(c) The solution set of the equation $x^2 + 2x + 3 = 0$

5) Graph the function f where $f(x) = x^2 - 5x + 3$, $x \in [0, 5]$ then from the graph find.

(a) The vertex point of the curve

(b) The minimum value of the function f.

(c) The two roots of the equation $x^2 - 5x + 3 = 0$

6) Graph the function f where $f(x) = x^2 + x - 2$, $x \in [-3, 2]$, then from the graph find.

(a) The vertex point of the curve.

(b) The symmetric axis.

(c) The two roots of the equation $x^2 + x - 2 = 0$

7) Graph the function f where $f(x) = -2(x + 1)^2$, $x \in [-5, 3]$ then from the graph solve the equation $x^2 + 2x + 1 = 0$

8) Graph the function f where $f(x) = x^2 - 2x$, $x \in [-2 , 4]$, then from the graph find :

(a) The vertex point of the curve

(b) The maximum or minimum value of the function f.

(c) The equation of the symmetric axis

(d) The solution set of the equation $f(x) = 0$

9) Graph the function f where $f(x) = x^2 - 1$, $x \in [-3 , 3]$, then from the graph find :

(a) The vertex point of the curve.

(b) The maximum or minimum value of the function f.

(c) The equation of the symmetric axis

(d) The solution set of the equation $f(x) = 0$

10) Graph the function f where $f(x) = 4 - x^2$, $x \in [-3 , 3]$, then from the graph find :

(a) The vertex point of the curve

(b) The maximum or minimum value of the function f

(c) The equation of the symmetric axis

(d) The two roots of the equation $x^2 = 4$

(3) Exercise on solving two equations in two variables one of first degree and the other of second degree.

First : complete the following:

1) The equation $xy = 3$ of .... degree.

2) The solution set of the two equations : $x = 1$, $x^2 + y^2 = 10$ is ....

3) If $x - y = 3$, $x^2 - y^2 = 6$, then $x + y =$ ....

4) The solution set of the two equations $x = 1$, $x^2 + y^2 = 1$ is ....

5) The solution set of the two equations : $x = 2$, $xy = 6$ is

6) If the sum of two positive numbers is 3, and the sum of their squares is 5, then the two numbers are ... , ...

7) If the sum of two positive numbers is 5, and their product is 6, then the two numbers are ... , ...

8) If the ratio between the perimeters of two squares is 1 : 2 then the ratios between their areas is

9) The area of the rectangle whose length is 3 cm. and its perimeter is 10cm equals

10) A square of side length 4cm, if this length increases by 3cm than its area increases by ... cm^2

Second : Choose the correct answer from given ones:

1- The degree of the equation $3x + 4y + xy = 5$ is ...

a) zero   b) first   c) second   d) third

2- One solution of the equation $x^2 - y^2 = 3$ in R may be ....

a) (1 , -2)   b) (-2 , 1)   c) (1, 2)   d) (-1 , -2)

3- The ordered pair that satisfies both of the two equations $xy = 2$, $x - y = 1$ is ...

a) (1 , 2)   b) (2, 1)   c) (1 , 1)   d) (2 , -1)

4) The solution set of the two equations : $x = y$, $xy = 1$ is ...

a) {(1, 1)}   b) {(-1 , -1)}

c) {(1 , -1)}   d) {(-1 , -1), (1,1)}

5) The solution set of the two equations $x - y = 0$, $xy = 9$ is ...

a) {(0 , 0)}   b) {(-3 , -3)}

c) {(3 , 3)}   d) {(-3 , -3), (3 , 3)}

6) One solution of the equation $x - y = 2$, $x^2 + y^2 = 20$ in R may be ... .

a) (-4 , 2) b) (2, -4) c) (3 , 1) d) (4 , 2)

7) If $x = y + 1$, $(x - y)^2 + y = 3$, then y equals ... .

a) zero b) 1 c) 2 d) 3

8) If $x = 1$, $x^2 + y^2 = 10$, then y equals ... .

a) -3 b) ± 3 c) 2 d) 3

9) If $a b = 3$, $ab^2 = 12$, then b equals ... .

a) 4 b) 2 c) -2 d) ± 2

10) If the difference between two numbers is 1 and the square of their sum is 25, then the two numbers are ... .

a) 1 , 2 b) 2 , 3 c) 3 , 4 d) 4, 5

Third: Find the solution set for each pair of the following equations.

1) $x + 1 = 0$, $x^2 + y^2 = 17$
2) $x - 2 = 0$, $x^2 + xy + y^2 = 7$
3) $x - y = 0$, $xy = 1$
4) $x + y = 0$, $2x^2 - y^2 = 4$
5) $x - 2y = 0$, $x^2 - y^2 = 3$
6) $x - y = 1$, $x^2 + y^2 = 25$
7) $y = x - 5$, $x^2 - 2xy = 16$
8) $y - x = 2$, $x^2 + xy - 4 = 0$
9) $x - 2y - 1 = 0$, $x^2 - xy = 0$
10) $Y + 2x = 7$, $2x^2 + x + 3y = 19$

Fourth : Applications:

1) If the sum of integer numbers is 3, and the sum of their squares is 5 find the two numbers
2) Two numbers one of them is the additive inverse of the other, and the sum of their squares is 2, find the numbers
3) If the difference between two numbers is 5, and their product is 36, then find the two numbers.
4) If the sum of two positive numbers is 9 and the difference between their squares is 27 find the two numbers
5) Find the number which is formed from two digits, if the units digit is twice the tens digit, and if the product of the two digits equals half the original number
6) The length of a rectangle is 3 more than its width, and its area is 28 cm^2. Find its perimeter
7) Find the two dimensions of a rectangle if its perimeter is 24 cm and its area is $35 cm^2$.
8) Find the two dimensions of a rectangle if its diagonal of length 5 cm, and its perimeter is 14cm
9) The hypotenuse of a right angled triangle is 13cm, and its perimeter is 30cm find the lengths of the other two sides
10) The difference between the lengths of the two rhombus's diagonals is 4cm. and its perimeter is 40 cm, find the length of each diagonal.

Model Answers Part (1)

(1) First complete :

1) (4,3)
2) 3rd
3) {(-1 , -2)}
4) {(-5 , 5)}
5) Ø
6) {(x , y) , y= 6-4x , (x , y) ∈ R x R}
7) a = 3
8) K ≠ 4

Second. choose:

1) c
2) a
3) a
4) a
5) b
6) d
7) d
8) d
9) c
10) a
11) a
12) c

Third: Find the S.S.

1) {(1 , -3)}
2) {(4 , 2)}
3) {(2 , 3)}
4){(2, 3)}
5) ϕ
6) {(2 , 5)}
7) {(-1 , 3)}
8) {(-2 , 3)}
9) {(2, -4)}
10) {(x, y), x= $\frac{3}{2}$y + $\frac{7}{2}$}

Fouth : Find the S.S.

1) {(3.5 , 3)}
2) {(2 , 7)}
3) {(2 , 3)}
4) {(2 , 2)}
5) {(5 , -0.2)}
6){(-1 , 1)}
7) {(1 , 1)}
8) {(6 , 3)}
9) {(3 , -1)}
10) {(2, 0)}

Fifth : Find the S.S.

1) {(-2 , 3)}
2) {(1 , 5)}
3) {(0 , 4)}
4) {(3 , -1)}
5){(-1 , 3)}

Sixth : Answer the questions :

1) $x + y = 63$ (1) , $x - y = 12$ (2) by adding (1) and (2)

$2x = 75 \rightarrow x = 37.5 \rightarrow y = 25.5$

2) $3x + 2y = 19$ (1)

$X + 3y = 16$ (2) $(x - 3)$

$-3x \quad 4y = -48$ (3)

By adding (1) and (3)

$-7y = -29 \rightarrow y = \frac{29}{7}$

$X = 16 - 3y = 16 - 3 \times \frac{29}{7} = \frac{25}{7}$

3) big no = x , mall no = y

$X + Y = 12$ (1) , $3y - 2x = 1$ (2)

$X = 12 - y$ by substituting into (2)

$3y - 2(12 \quad y) = 1$

$3y - 24 + 2y = 1 \rightarrow 5y = 25 \rightarrow y = 5$

$X = 12 - 5 = 7$

4) Let the rational no = $\frac{x}{y}$

$\frac{x \quad 3}{y - 3} = \frac{5}{6}$

$6x - 18 = 5y - 15$

$6x - 5y = 3$ (1) $(x\ 13)$

$\frac{x+5}{y+5} = \frac{13}{14}$

$13y + 65 = 14x + 70$

$14x - 13y = -5$ (2) $(x\ 5)$

$$\left.\begin{array}{l} 78x - 65y - 39 \\ 70x - 65y = -25 \end{array}\right\} \text{By subtracting}$$

$8x = 64 \longrightarrow x = 8$

$6 \times 8 - 5y = 3$

$48 - 5y = 3 \rightarrow -5y = -45$

$\rightarrow y = 9$

The no. is $\frac{8}{9}$

5) let the unit digit = x

the tens digit = y

$x + y = 11$ (1) $(x - 2)$

$2x - 3y = 2$ (2)

$-2x - 2y = -22$ by adding

$-5y = -20 \longrightarrow y = 4$

$X = 11 - 4 = 7$ the no. is 47

6) x= unit , y = tens.

$x + y = 5$ (1)

The original No. = x + 10y

The no. after exchanging = y + 10x

$(x + 10y) - (y + 10x) = 9$

$X + 10y - y - 10x = 9$

$-9x + 9y = 9 \longleftrightarrow -x + y = 1$ (2)

By adding (1) and (2)

$2y = 6 \longrightarrow y = 3$

$X = 5 - 3 = 2$

The original no. 32

1) man's age = x , son's ago = y

6 years ago : $x - 6$, $y - 6$

$X - 6 = 6(y - 6) = 6y - 36$

$x - 6y = -30$ (1)

after 10 years

$x + 10$, $y + 10$

$x + 10 = 2(y + 10) = 2y + 20$

$x - 2y = 10$ (2)

by subtracting (2) from (1)

$-4y = -40 \longrightarrow y = 10$

$X = 10 + 2y = 10 + 20 = 30$

The man's age = 30 son's ago = 10

8) $L = x$, $w = y$

$X - y = 3$ (1) $\longrightarrow x = y + 3$

$4y - 2x = 2$ (2)

$4y - 2(y + 3) = 2 \rightarrow 4y - 2y - 6 = 2$

$2y = 8 \rightarrow y = 4$ cm .

9) $L + w = \frac{p}{2} = \frac{32}{2} = 16$

$x + y = 16$ (1)

$L - 1$, $w + 3$

$x - 1 = y + 3 \longrightarrow x = y + 4$ (2)

$y + 4 + y = 16 \longrightarrow 2y = 12 \rightarrow y = 6$ cm

$x = 16 - 6 = 10$ cm

area of square $= S^2 = 9^2 = 81$ cm^2

10) $x + y = 90°$ (1)

$x - y = 30°$ (2) by adding

$2x = 120° \longrightarrow x = 60°$

$y = 90° - 60° = 30°$

11) Exercise on solving and degree equations .

1) b 2)b 3) c 4) d 5) b

Second: formula

$$X = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(1) $a = 1$, $b = -2$, $c = -4$

$$X = \frac{(\ 2) \pm \sqrt{(\ 2)^2 \ \ 4(1)(\ 4)}}{2(1)}$$

$$X = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm \sqrt{4 \times 5}}{2}$$

$$X = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$

$1 + \sqrt{5}$, $1 - \sqrt{5}$

S.S. = {3.24 , - 1.24}

(2) $x^2 = 2x + 12$, $x^2 \ \ 2x - 12 = 0$

S.S. = {4.6 , -2.6}

(3) $x^2 - 2x + 1 - 10 = 0$

$x^2 - 2x - 9 = 0$

S.S. = {3. 16 , -2.16}

(4) $x^2 - 2x - 6 = 0$,

S.S. = {3.65 , -1.65}

(5) $x^2 - 6x + 9 \ \ 3x + 9 + 1 = 0$

$x^2 - 9x + 19 = 0$

S.S. = {5.62 , 3.38}

(6) $1 - \frac{2}{x} = \frac{2}{x^2}$ $(x\ x^2)$

$x^2 - 2x = 2 \rightarrow x^2 - 2x - 2 = 0$

S.S. = {2.73 , - 0.73}

(7) S.S. = {1.33}

(8) S.S. = $\varnothing$

(9) $x + \frac{4}{x} + 1 = 0$ (x X)

$x^2 + 4 + x = 0 \rightarrow x^2 + x + 4 = 0$

S.S. = $\varnothing$

(10) let 1 $y^2 = y$

$y^2 + 2y - 100 = 0$

$y = \frac{-b \pm \sqrt{4ac}}{2a}$

$= \frac{2 \pm \sqrt{4 \quad (4X\ 1X \quad 1)}}{2x1}$

$= \frac{-2 \pm \sqrt{8}}{2}$

$= \frac{-2 \pm 2\sqrt{2}}{2}$

$= -1 \pm \sqrt{2}$

$\rightarrow Y = -1 \pm \sqrt{2}$

$\rightarrow x^2 = -1 + \sqrt{2}$ or $x2 = -1 - \sqrt{2}$ refused

$x^2 = \sqrt{2} - 1$

Third Answer the following questions:

Draw by yourself

Exercises on solving two equations (1st and 2nd degree)

First Complete :

(1) 2nd (2) {(1 , 3) , (1 , -3)} (3) 2

(4) {(1 , 0)} (5) {(2 , 3)} (6) 1 , 2

(7) 2 , 3 (8) $P_1 : P_2 = S_1 : S_2 = 1 : 2$ areas $S_1^2 : S_2^2 = 1 : 4$

(9) $L+W = \frac{P}{2}$ → $3 + w = 5$ → $w = 2$ area = 2x3 = 6cm^2

(10) area = 4 x 4 = 16 cm^2

area = $(4 + 3)^2 = 7^2 = 49$ cm^2

$49 - 16 = 33$

area increases by 33 cm^2

Second choose:

(1) c (2) b (3) b (4) d (5) d

(6) d (7) c (8) b (9) a (10) b 2,3

Third: Find the S S.:

(1) $x = -1$ → $(-1)^2 + y^2 = 17$

$y^2 = 16 \rightarrow y = \pm\sqrt{16} = \pm 4$

S.S. = {(-1 , 4) , (-1 , -4)}

(2) S.S. = {(2 , 1) , (2, -3)}

(3) $x = y$ → $x^2 = 1$ → $x = \pm\sqrt{1} = \pm 1$ → $y = \pm 1$

S.S. = {(1 , 1) , (-1 , -1)}

(4) $x = -y$ → $2(-y)^2 - y^2 = 4$

$2y^2 - y^2 = 4$ → $y^2 = 4$ → $y = \pm 2$

At $y = 2 \rightarrow x = -2$, at $y = -2 \rightarrow x = 2$

S.S.{(2, -2) , (-2 , 2)}

(5) S.S. = {(2 , 1), (-2 , -1)}

(6) S.S. = {(4 , 3), (-3 , -4)}

(7) S.S. = {(2 , -3), (8 , 3)}

(8) S.S. = {(-2 , 0), (1 , 3)}

(9) S.S. = {(0 , $-\frac{1}{2}$), (-1 , -1)}

(10) S.S. = {($\frac{1}{2}$, 6), (2 , 3)}

Fourth : Applications:

(1) $x + y = 3$ (1)

$x^2 + y^2 = 5$ (2)

$x = 3 - y$

$(3 - y)^2 + y^2 = 5$

$9 - 6y + y^2 + y^2 = 5$

$2y^2 - 6y + 4 = 0$ (÷2)

$y^2 - 3y + 2 = 0$

$(y - 2)(y - 1) = 0$

$y - 2 = 0 \rightarrow y = 2$

$x = 3 - 2 = 1$

or $y - 1 = 0 \rightarrow y = 1$

$x = 3 - 1 = 2$ the two no. are 1 and 2

(2) first no. = x , 2nd = -x

Or 1st = x , 2nd = y

Then $x = -y$ (1)

$x^2 + (-y)^2 = 2$

$x^2 + y^2 = 2$ (2)

$(-y)^2 + y^2 = 2$

$2y^2 = 2 \rightarrow y^2 = 1 \rightarrow y = \pm 1$

$X = 1$ when $y = -1$

$X = -1$ when $y = 1$

the two nos. are 1 and -1

(3) $x - y = 5$ (1) $\rightarrow x = y + 5$

$xy = 36$ (2)

$y(y + 5) = 36$

$y^2 + 5y - 36 = 0$

$(y - 4)(y + 9) = 0$

$y = 4 \rightarrow x = 9$

Or $y = -9 \rightarrow x = -4$

the two numbers are 4 , 9 or - 4 , -9

(4) $x + y = 9$ (1)

$x^2 - y^2 = 27$ (2)

$x = 9 - y$

$(9 - y)^2 - y^2 = 27$

$81 - 18y + y^2 - y^2 - 27 = 0$

$54 - 18y = 0$

$y = 3 \rightarrow x = 9 - 3 = 6$

the two nos. are 3 and 6

(5) Original no. $x + 10y$

$x \rightarrow$ units , $y \rightarrow$ tens

$x = 2y$ (1)

$x \times y = \frac{(x + 10y)}{2}$

$2xy = x + 10y$

$X + 10y - 2xy = 0$ (2)

$2y + 10y - 2y\,(2y) = 0$

$12y - 4y^2 = 0 \quad \rightarrow -4y^2 + 12y = 0$

$-4y\,(y - 3) = 0$

$y = 0$ or $y = 3$

$X = 0$ refused $\quad x = 6$

The no. is 36

(6) $L = x \quad , w = y$

$X - y = 3 \quad (1) \rightarrow \quad x = y + 3$

$Xy = 28$

$y\,(y + 3) = 28$

$y^2 + 3y \quad 28 = 0$

$(y + 7)\,(y - 4) = 0$

$y = -7$ or $y = 4$

Refused $\quad x = 4 + 3 = 7$

$P = 2\,(L + w) = 2\,(7 + 4) = 22$ cm.

(7) $L = x \quad , y = w$

$\therefore \quad x + y = \frac{p}{2} = 12 \quad (1)$

$Xy = 35$

$X = 12 - y$

$Y\,(12 - y) = 35$

$12y - y^2 - 35 = 0$

$-y^2 + 12y \quad 35 = 0 \quad (\div -1)$

$y^2 - 12y + 35 = 0$

$(y \quad 7)\,(y \quad 5) = 0$

$y = 7$ or $y = 5$

$x = 12 - 7 = 5$ $\qquad$ $x = 12 - 5 = 7$

the two dimensions are 7 , 5

(8) $L = x$, $w = y$

$\because$ the p. = 14

$\therefore x + y = \frac{14}{2} = 7$ $\qquad$ (1)

$\because$ Δ ABC is right angled at B

$\therefore x^2 + y^2 = (5)^2$ (Pythagoras)

$x^2 + y^2 = 25$ $\qquad$ (2)

$x = 7 - y$

$(7 - y)^2 + y^2 = 25$

$49 - 14y + y^2 + y^2 - 25 = 0$

$2y^2 - 14y + 24 = 0$ $\qquad$ $(\div 2)$

$y^2 - 7y + 12 = 0$

$(y - 3)(y - 4) = 0$

$y = 3$ or $y = 4$

the two dimensions are 3 and 4

(9) $x^2 + y^2 = 169$ $\qquad$ (1)

$x + y + 13 = 30$

$x + y = 17$ $\qquad$ (2)

$x = 17 - y$

$(17 - y)^2 + y^2 - 169 = 0$

$y^2 + 289 - 34y + y^2 - 169 = 0$

$2y^2 - 34y + 120 = 0$ $\qquad$ $(\div 2)$

$y^2 - 17y + 60 = 0$

$(y-12)(y-5)=0$

$y=12 \quad \rightarrow \quad x=5$

Or $y=5 \quad \rightarrow x=12$

The other two sides are of length 12 cm and 5cm

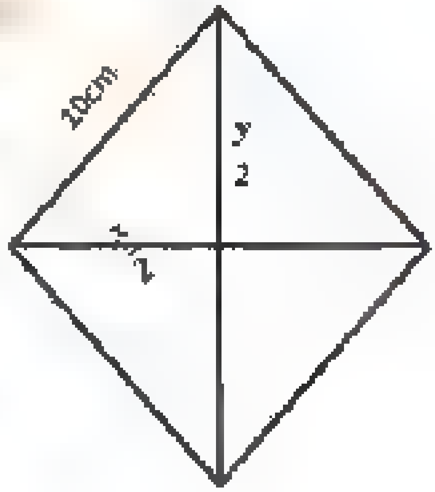
(10) let one of the two diagonals of = x and the other = y

- Half the diagonals will be $\frac{x}{2}$ and $\frac{y}{2}$

∵ the p. of Rhombus = 40 cm then each S= 40 ÷ 4 = 10cm

∵ the two diagonals are peroendicular

∴ $(\frac{x}{2})^2+(\frac{y}{2})^2=(10)^2$



$\frac{x^2}{4}+\frac{y^2}{4}=100 \qquad (\times 4)$

$x^2+y^2=400 \qquad (1)$

$x-y=4 \quad (2) \quad \rightarrow \quad x=y+4$

$(y+4)^2+y^2-400=0$

$y^2+8y+16+y^2-400=0$

$2y^2+8y-384=0 \qquad (\div 2)$

$y^2+4y-192=0$

$(y-16)(y+12)=0$

$y=16cm \;\; (x=16+4=20cm) \quad$ or $\quad y=-12$ (refused)

The algebraic fractions and operations

Questions Part (2)

First: Complete the following:

1) The domain of the function f where $f(x) = \frac{x+2}{x-1}$ is ....

2) The domain of the function f where $f(x) = \frac{x^2 - x}{x^2 - 2x - 3}$ is ....

3) The domain of the function f where $f(x) = \frac{x+2}{5x}$ is ....

4) The domain of the function f where $f(x) = \frac{x^2+7}{x^2+4}$ is ....

5) The common domain of the two functions $f_1(x) = \frac{x+1}{x}$, $f_2(x) = \frac{x-3}{x^2 - 5x + 6}$ is ......

6) The simplest form of the algebraic fraction $\frac{x-3}{x^2 - 5x + 6}$ is ....

7) If $N_1(x) = \frac{5x}{5x^2+20}$, $N_2(x) = \frac{x}{x^2+4}$, then $N_1 = N_2$ in the domain ....

8) If $N_1 = \frac{x+2}{x^2-4}$, $N_2 = \frac{x+5}{(x+5)(x-2)}$, then $N_1 = N_2$ in the domain ....

9) The set of zeroes of f where $f(x) = 5 - x$ is ....

10) If $N(x) = \frac{x^2-9}{x-2}$, then $Z(N) =$ ....

11) The set of zeroes of f where $f(x) = \frac{x-2}{x^2-4}$ is ....

12) The set of zeroes of f where $f(x) = x^2 - 25$ is ....

13) The function $f(x) = \frac{x-5}{x-2}$ does not exist at $x =$ ......

14) If $N(x) = \frac{1}{x+2} + \frac{1}{x-2}$, then its simplest form is .... , and its domain is ......

15) The domain of the additive inverse of the fraction $n(x) = \frac{2}{x-1}$ is ....

Second : Choose the correct answer from the given ones

(1) The function f where $f(x) = \frac{x-2}{x^3+27}$, then the domain of its multiplicative inverse is ....

(a) R – {2}   (b) R – {–3 , 2}   (c) R – {2 , –3,3}   (d) R – {3 , –3}

(2) If the function f where $f(x) = \frac{x^2-9}{x}$, has a multiplicative inverse, then their common domain is ....

(a) R – {0}   (b) R – {0 , 3}   (c) R – {0,3 , –3}   (d) R

(3) If $N(x) = \frac{x-1}{x-2}$, then the domain of $N^{-1}(x)$ ....

(a) R   (b) R – {1}   (c) R – {2}   (d) R – {1,2}

(4) The function f where $f(x) = \frac{x-2}{x-5}$ has a multiplicative inverse if its domain is ....

(a) R   (b) R – {5}   (c) R – {2}   (d) R – {2,5}

(5) The function f where $f(x) = \frac{x-2}{x-5}$ has a multiplicative inverse if the domain is ....

(a) R – {2}   (b) R – {5}   (c) R – {-2 , 2}   (d) R – {0,1}

(6) If $N(x) = \frac{1}{x} - \frac{3}{x}$, then $N^{-1}(x) =$ ....

(a) $x - \frac{x}{3}$ (b) $\frac{2}{x}$ (c) $\frac{-x}{2}$ (d) $\frac{x}{2}$

(7) The domain of the function f where $f(x) = \frac{x(x+2)}{x^2-4}$ is ....

(a) R   (b) R – {-2 , 2}   (c) R – {2,0}   (d) R – {2}

(8) The domain of the function f where $f(x) = \frac{x-3}{2}$ is ....

(a) R   (b) R – {0}   (c) R – {-1 , 0}   (d) R – {0 , 1}

(9) The domain of the function f where $f(x) = \frac{x-2}{3(x+1)}$ is ....

(a) R   (b) R – {1}   (c) R – {-1 , 3}   (d) R – {-1}

(10) The domain of the function n where $n(x) = \frac{x-1}{x+2} + \frac{x-2}{x+1}$ is ....

(a) R – {-1}   (b) R – {-2}   (c) R – {-1 , -2}   (d) R – {-1 , -2 , 1 , 2}

(11) If $n(x) = \frac{2x}{x^2-x+2}$, then n(-1) ....

(a) $\frac{-1}{2}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) 2

12) The function f where $f(x) = \frac{x+2}{x-2}$, then the domain of its multiplicative inverse is ......

(a) R (b) R − {2} (c) R − {-2} (d) R − {-2 , 2}

(13) The domain of the function n where $n(x) = \frac{x-2}{x+3} - \frac{3x}{x-1}$ is

(a) R − {0 , 2} (b) R − {-3 , 1} (c) R − {2 , 3} (d) R − {1 , 2}

14) The simplest form of the function $n(x) = \frac{x}{x-3} + \frac{3x}{x^2-9}$ is

(a) $\frac{x}{x-3}$ (b) $\frac{x}{x+3}$ (c) $\frac{x+3}{3}$ d) $\frac{x-3}{3}$

(15) The domain of the multiplicative inverse of the fraction $\frac{x+7}{x-2}$ is ......

(a) R (b) R − {2} (c) R − {-7} d) R − {-7 , 2}

(16) The additive inverse of the fraction $\frac{3}{x^2+1}$ is

(a) $\frac{-3}{x^2+1}$ (b) $\frac{x^2+1}{3}$ (c) $\frac{-x^2+1}{3}$ d) $\frac{3}{x^2-1}$

(17) If $f(x) = \frac{x^2-9}{x+b}$, f(4) = 1, then b = ...

(a) -7 (b) 7 (c) 3 d) -3

(18) If $N(x) = \frac{x-2}{x^2-x-6}$, then the domain of $N^{-1}(x)$ ....

(a) R − {2} (b) R − {-2 , 3} (c) R − {-2 , 2} d) R − {-2 , 2 , 3}

19) The simplest form of the function $n(x) = \frac{x+1}{x-1} + \frac{1-x}{x-1}$, $x \neq 1$ is ....

(a) zero (b) $\frac{2}{2x-2}$ (c) $\frac{2}{x-1}$ d) $\frac{2}{(x-1)^2}$

20) The set of zeroes of f where $f(x) = (x-1)^2(x+2)$ is ......

(a) {1 , 2} (b) {1 , -2} (c) {-1 , 2} (d) {-1 , -2}

Third: Answer the following questions

(1) Simplify each of the two algebraic fractions $n_1(x) = \frac{x^2-1}{x^2-x}$, $n_2(x) = \frac{2x-6}{x^2-5x+6}$

(2) Simplify the function n where $n(x) = \frac{3x}{x^2-2x} - \frac{12}{x^2-4}$ showing its domain.

(3) Simplify the function n where $n(x) = \frac{x^2-1}{x^2+3x+2} + \frac{x^2-x}{x^2+2x}$, showing its domain.

(4) Find n in its Simplest form wher $n(x) = \frac{x}{4} + \frac{2}{x+2}$ showing its domain.

(5) If the domain of the function n where $n(x) = \frac{b}{x} + \frac{9}{x + a}$ is $R - \{0 , 4\}$, $n(5) = 2$.

Find the value of a and b

6) Find n in its Simplest form where $n(x) = \frac{3x^2 + 6x}{x^2 - 4} \times \frac{x - 2}{2x + 6}$, showing its domain

(7) Find n in its Simplest form where $n(x) = \frac{x + 3}{(x - 2)(x + 7)} + \frac{x^2 + 3x}{2x + 14}$, showing its domain.

(8) If $n_1(x) = \frac{x^3}{x^4 - x^3}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$, prove that $n_1 = n_2$

(9) Find n in its simplest form, showing its domain, where

1) $n(x) = \frac{x}{x + 1} + \frac{2x^2}{x^2 - x}$,

2) $n(x) = \frac{x - 1}{x^2 - 1} + \frac{x^2 - 5x}{x^2 - 4x - 5}$

(10) Find f in its Simplest form, where

$f(x) = \frac{3x^2 - 6x}{x^2 - 4} \times \frac{x^2 + 3x + 2}{x^2 + x}$

(11) Find the common domain of f_1 , f_2 to be equal such that:

$f_1(x) = \frac{x^2 + x - 12}{x^2 + 5x + 4}$, $f_2(x) = \frac{x^2 - 2x - 3}{x^2 + 2x + 1}$

(12) Find n in its simplest form, showing its domain, where

1) $f(x) = \frac{x^2 - x}{x^2 - 1} + \frac{x - 5}{x^2 - 6x + 5}$,

2) $f(x) = \frac{x^3 - 1}{x^2 - 2x + 1} \times \frac{2x - 2}{x^2 + x + 1}$

13) If $f(x) = \frac{x^2 - 49}{x^3 - 8} + \frac{x + 7}{x - 2}$ Find f in its Simplest form, showing its domain.

Then calculate $f(1)$.

(14) Find the common domain of f_1 , f_2 to be equal such that

$f_1(x) = \frac{x^2 + 3x + 2}{x^2 - 4}$, $f_2(x) = \frac{x^2 - 1}{x^2 - 3x + 2}$

(15) Find n in its simplest form, showing its domain, where

a) $n(x) = \frac{3x}{x^2 - x - 2} + \frac{x - 1}{1 - x^2}$,

b) $n(x) = \frac{x}{x - 2} + \frac{x + 3}{x^2 - x - 2}$

General Exercise on The Probability

First : Complete the following:

(1) The two events are said to be mutually exclusive if $A \cap B = ...$

(2) If the probability that the event A occurs is 75%, the probability of non occurrence of this event is ......

(3) If A is an event, $P(A) = 0$, then A is ....

(4) If A' is the complement event of A, then $A \cup A' = ...$, $A \cap A' = ...$

(5) The probability of the sure event equals ....

(6) The probability of the impossible event equals ....

(7) When a regular die is tossed once, then the probability of getting an even number is ......

(8) When a regular coin is tossed once, then the probability of getting a head is ......

(9) If A, B are two mutually exclusive events, $P(A) = 0.2$ and $P(B) = 0.3$, then $P(A \cup B) = ...$

(10) If A , B are two mutually exclusive events of a random experiment, then $P(A \cap B) =$ ....

(11) If $A \subset S$ of a random experiment, $P(A) = P(A')$, then $P(A) =$ ......

(12) If A , B are two mutually exclusive events of a random experiment, $P(A) = \frac{1}{4}$, $P(A \cup B) = \frac{5}{12}$, then $P(B) = ...$

Second: Choose the correct answer from the given ones

(1) If a regular die is tossed once, the probability of appearance of a number less than 3 equals ......

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) $\frac{2}{3}$

(2) If a bag contains 4 white balls, 6 red balls if one ball is drawn randomly, then the probability that this ball is red equals ......

(a) $\frac{1}{5}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$

(3) If the probability that a student in preparatory final exam is succeeded equals 85%, then the probability that he fail is ......

(a) 0.015 (b) $\frac{3}{20}$ (c) $\frac{17}{20}$ (d) 0.85

(4) If the probability that a the Egyptian team may win a football in the African Cup of Nations is 0.318, then the probability of non winning is ......

(a) 1 (b) zero (c) 0.862 (d) 0.682

(5) If a bag contains a number of identical green and blue balls, if one ball is drawn randomly, the number of green balls is 5 while the probability that the drawn ball is blue equals $\frac{2}{3}$, then the number of blue balls equals ......

(a) 10 (b) 12 (c) 15 (d) 20

(6) If $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{8}$, then $P(A \cup B) = ...$

(a) $\frac{5}{8}$ (b) $\frac{17}{24}$ (c) $\frac{1}{6}$ (d) $\frac{13}{24}$

(7) If $P(A) = 0.2$, $P(B) = 0.6$ and $P(A \cap B) = 0.3$, then $P(A \cup B) = ...$

(a) 0.5 (b) 0.62 (c) 5 (d) 0.13

(8) If A, B are two mutually exclusive events, $P(A) = 0.5$ and $P(A \cup B) = 0.8$, then $P(B) = ...$

(a) 0.03 (b) 0.3 (c) 0.5 (d) 0.13

(9) A card is drawn randomly from 20 identical cards numbered from 1 to 20, then the probability that the number of the drawn card multiple of 7 is ......

(a) 10% (b) 15% (c) 20% (d) 25%

(10) If A, B are two events in a random experiment and $A \subset B$, then $P(A - B) = ...$

(a) zero (b) $P(A) - P(B)$ (c) $P(B) - P(A)$ (d) $P(A)$

Third : Answer the following questions

(1) A card is drawn randomly from 20 identical cards numbered from 1 to 20, calculate the probability that the number on the drawn card is:

(a) A number divisible by 5

(b) A number divisible by 4

(c) A number divisible by 5 and divisible by 4

(d) A number divisible by 5 or divisible by 4

(2) If A, B are two events in a random experiment and if $P(A) = 0.2$, $P(B) = 0.6$ and $P(A \cup B) = 0.5$, find $P(A \cap B)$.

(3) If a bag contains 21 identical balls, 8 white balls, 6 red balls and the rest are black. If one ball is drawn randomly, find the probability that this ball is :

(a) White (b) Not black (c) Red or black

(4) A box contains 30 identical cards numbered from 1 to 30 one card of them is drawn randomly calculate the probability that the number of the drawn card is :

(a) Odd and divisible by 5 (b) Prime or divisible by 7

(5) During a training football clubs a player hits 24 penalty kick including 21 goals another player hitting 27 including 24 goals. Who of the two players can be chosen to play the penalty ? Explain your answer.

(6) One of the companies producing refrigerators Conducted questionnaire about the production of refrigerators on a set of 500 women to find out their view on the refrigerators Sizes results were as follows:

Size in foot	6	10	12	14	16	Total
frequency	25	90	165	130	90	500

If a woman is chosen randomly, what the probability that the size favorite of the refrigerator is

(a) 6 foot (b) 10 foot (c) 12 foot (d) 14 foot (e) 16 foot

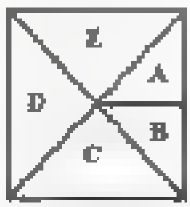
(7) A card is drawn randomly from 50 identical cards numbered from 1 to 50, find the probability that the number of the drawn card is.

(a) divisible by 10

(b) divisible by 11

(c) divisible by 10 or divisible by 11

(d) Not complete square



(8) The player should be able to release the arrow without located on the line between any two of the target areas.

1. what is the probability that the arrow hits the area D ?
2. what is the probability that the arrow hits the area A ?
3. what is the probability that the arrow hits the area B or C ?

(9) A classroom consists of 40 students, 30 of them succeeded in math , 24 in science and 20 in both math. and science. if a student is chosen randomly. Find the probability that this student is :

(a) Succeeded in math.

(b) Succeeded in science.

(c) Fail in math

(d) Succeeded in math. or science.

(10) A classroom consists of 42 students, 20 of them play football, 8 play basketball and the other students play other sports, if a student is chosen randomly. Find:

First : the probability that this student is playing football

Second : if this class is chosen from all classes and the number of the total student is 600, find the number of students who play other sports.

(11) A box contains 15 identical ball, 6 of them are red numbered from 1 to 6 and 9 green numbered from 7 to 15 one ball of them is drawn randomly. Find the probability that:

(a) The drawn ball is red or has on odd number.

(b) The drawn ball is green and has an even number.

(12) The opposite table shows that 120 visitors visited the exhibition, if one of them is chosen randomly. Find the probability that :

(a) The visitor is a female.

(b) The visitor is a foreign.

(c) The visitor is a male or a foreign.

	Arabic	Foreign	Total
Male	48	16	64
Female	32	24	56
Total	80	40	120

Model Answers Part (2)

(1) First complete :

1) R – {1} 2) R- {-3 , 1} 3) R- {0} 4) R

5) R- {0 , 2 , 3} 6) $\frac{1}{x-2}$ 7) R 8) R- {2 , -2 , -5}

9) {5} 10) R- {2} 11) {2} 12) {5 , -5}

13) 2 14) R- {2 , -2} 15) R – {1 }

(2) Choose:

1) b 2) c 3) d 4) d 5) R-{ 2 , 5} 6) c

7) b 8) a 9) d 10) c 11) a 12) d

13) b 14) c 15) d 16) a 17) c 18) d

19) c 20) b

(3) Answer the questions:

(1) $n_1(x) = \frac{(x-1)(x+1)}{x(x-1)} = \frac{x+1}{x}$

$n_2(x) = \frac{2(x-3)}{(x-2)(x-3)} = \frac{2}{x-2}$

(2) $n(x) = \frac{3x}{x(x-2)} - \frac{12}{(x-2)(x+2)}$

D(n)=R – {0 , 2 , -2}

$n(x) = \frac{3}{x-2} - \frac{12}{(x-2)(x+2)}$

$= \frac{3(x+2)}{(x-2)(x+2)} - \frac{12}{(x-2)(x+2)}$

$= \frac{3x-10}{(x-2)(x+2)}$

(3) $n(x) = \frac{(x-1)(x+1)}{(x+2)(x+1)} \div \frac{x(x-1)}{x(x+2)}$

$D(n) = R - \{0, 1, -2, -1\}$

$n(x) = \frac{(x-1)}{(x+2)} \times \frac{(x+2)}{(x-1)} = 1$

(4) $n(x) = \frac{x}{4} + \frac{-2}{x+2}$

$D(n) = R - \{-2\}$

$n(x) = \frac{x(x+2)}{4(x+2)} + \frac{-2 \times 4}{4(x+2)}$

$= \frac{x^2+2x-8}{4(x+2)} = \frac{(x+4)(x-2)}{4(x+2)}$

(5) $\because$ the domain is $R - \{0, 4\}$

$\therefore x = 0$ or $x = 4$

$x + a = 0 \rightarrow 4 + a = 0 \rightarrow a = -4$ if $n(5) = 2$

$\therefore n(5) = \frac{b}{5} + \frac{9}{5+(-4)} = 2$

$\frac{b}{5} + 9 = 2 \rightarrow \frac{b}{5} = -7 \rightarrow b = -35$

(6) $n(x) = \frac{3x(x+2)}{(x-2)(x+2)} \times \frac{x-2}{2(x+3)}$

$D(n) = R - \{2, -2, -3\}$

$n(x) = \frac{3x}{2(x+3)}$

(7) $n(x) = \frac{x+3}{(x-2)(x+7)} \div \frac{x(x+3)}{2(x+7)}$

$D(n) = R - \{0, -3, 7, -7, 2\}$

$n(x) = \frac{(x+3)}{(x-2)(x+7)} \times \frac{2(x+7)}{x(x+3)}$

$n(x) = \frac{2}{x(x-2)}$

(8) $n_1(x) = \frac{x^2}{x^2(x-1)} = \frac{1}{x-1}$ .... (1)

$D_1(n_1) = R - \{0 , 1\}$

$n_2(x) = \frac{x(x^2+x+1)}{x(x^3-1)}$

$= \frac{x(x^2+x+1)}{x(x-1)(x^2+x+1)} = \frac{1}{x-1}$ .... (2)

$D_2(n_2) = R - \{0 , 1\}$

$\because (1) = (2) , \because D_1 = D_2$

$\therefore n_1 = n_2$

(9) $n(x) = \frac{x}{x+1} + \frac{2x^2}{x(x-1)(x+1)}$

$D(n) = R - \{-1 , 0 , 1\}$

$N(x) = \frac{x(x-1)}{(x+1)(x-1)} + \frac{2x}{(x-1)(x+1)}$

$= \frac{x^2-x+2x}{(x+1)(x-1)} = \frac{x^2+x}{(x+1)(x-1)}$

$= \frac{x(x+1)}{(x+1)(x-1)} = \frac{x}{x-1}$

(2) $n(x) \frac{x-1}{(x-1)(x+1)} \div \frac{x(x-5)}{(x-5)(x+1)}$

$D(n) = R - \{0 , 5 , -1 , 1\}$

$n(x) = \frac{1}{x+1} \times \frac{x+1}{x} = \frac{1}{x}$

(10) $f(x) = \frac{3x(x-2)}{(x-2)(x+2)} \times \frac{(x+2)(x+1)}{x(x+1)}$

$D(f) = R - \{2 , -2 , 0 , -1\}$

$F(x) = 3$

(11) $f_1(x) = \frac{(x-3)(x+4)}{(x+4)(x+1)} - \frac{x-3}{x+1}$

$D(f_1) = R - \{-4 , -1\}$

$f_2(x) = \frac{(x-3)(x+4)}{(x+1)(x+1)} - \frac{x-3}{x+1}$

$D(f_2) = R - \{-1 , 1\}$

$f_1 = f_2$ when $x \in R - \{-4 , -1 , 1\}$

(12)(1) $f(x) = \frac{x(x-1)}{(x-1)(x+1)} + \frac{x-5}{(x-5)(x-1)}$

$D(f) = R - \{1 , -1 , 5\}$

$f(x) = \frac{x}{x+1} + \frac{1}{x-1}$

$= \frac{x(x-1)}{(x+1)(x-1)} + \frac{x+1}{(x-1)(x+1)}$

$= \frac{x^2-x+x+1}{(x+1)(x-1)} = \frac{x^2+1}{(x+1)(x-1)}$

(2) $f(x) = \frac{(x-1)(x^2+x+1)}{(x^2-2x+1)} \times \frac{2(x-1)}{(x^2+x+1)}$

$D(f) = R.$

$f(x) = \frac{2(x-1)^2}{x^2-2x+1}$

(13) $f(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \div \frac{x+7}{x-2}$

$D(f) = R - \{2 , -7\}$

$f(x) = \frac{(x-7)(x+7)}{(x-2)(x^2+2x+4)} \times \frac{x-2}{x+7}$

$f(x) = \frac{x-7}{x^2+2x+4}$

(14) $f_1(x) = \frac{(x+2)(x+1)}{(x-2)(x+2)} - \frac{x+1}{x-2}$

$D(f_1) = R - \{2 , -2\}$

$f_2(x) = \frac{(x-1)(x+1)}{(x-2)(x-1)} - \frac{x+1}{x-2}$

$D(f_2) = R - \{2, 1\}$

$f_1 + f_2$ when $x \in R - \{2, -2, 1\}$

(15) a) $n(x) = \frac{3x}{(x-2)(x+1)} - \frac{x-1}{x^2-1}$

$= \frac{3x}{(x-2)(x+1)} - \frac{x-1}{(x-1)(x+1)}$

$n(x) = \frac{3x}{(x-2)(x+1)} - \frac{x-1}{(x-1)(x+1)}$

$D(n) = R - \{2, -1, 1\}$

$n(x) = \frac{3x}{(x-2)(x+1)} - \frac{x-2}{(x-2)(x+1)}$

$= \frac{3x-x+2}{(x-2)(x+1)} = \frac{2(x+1)}{(x-2)(x+1)}$

$n(x) = \frac{2}{x-2}$

b) $n(x) = \frac{x}{x-2} \div \frac{x+3}{(x-2)(x+1)}$

$D(n) = R - \{-3, -1, 2\}$

$n(x) = \frac{x}{x-2} \times \frac{(x-2)(x+1)}{x+3}$

$= \frac{x(x+1)}{x+3}$

General exercise on the probability

First : Complete:

1) $\varnothing$

2) 25%

3) impossible event

4) 5 , $\varnothing$

5) 1

6) zero

7) $\frac{1}{2}$

8) $\frac{1}{2}$

9) $0.2 + 0.3 = 0.5$

10) zero

11) 1

12) $P(B) = P(A \cup B) - P(A) = \frac{5}{12} - \frac{1}{4} = \frac{1}{6}$

Second:- Choose:

1) b	2) c	3) b	4) d	5) a
6) b	7) a	8) b	9) a	10) a

Third:

1) a) $\frac{4}{20} = \frac{1}{5}$ {5 , 10 , 15 , 20}

b) { 4 , 8 , 12 , 16, 20}

$\frac{4}{20} = \frac{1}{4}$

c) {20} $p = \frac{1}{20}$

d) {5 , 10 , 15, 20 , 4 , 8 , 16}

$p = \frac{7}{20}$

2) P (A ∪ B) = P (A) + P (B). P(A ∩ B)

$0.5 = 0.2 + 0.6 - P(A \cap B)$, $P(A \cap B) = 0.3$

3) a) $\frac{8}{21}$ b) $\frac{14}{21} = \frac{2}{3}$ c) $\frac{13}{21}$

4) a) {5 ,15 , 20} $P = \frac{3}{30} = \frac{1}{10}$ b) $\frac{13}{30}$

5) $\frac{21}{24} = \frac{7}{8}$, $\frac{24}{27} = \frac{8}{9}$

$\frac{7}{8} < \frac{8}{9}$, then 2nd player is better.

6) a) $\frac{25}{500} = \frac{1}{2}$ b) $\frac{90}{500} = \frac{9}{50}$

c) $\frac{165}{500} = \frac{33}{100}$ d) $\frac{130}{500} = \frac{13}{50}$

e) $\frac{90}{500} = \frac{4}{50}$

7) a) $\frac{1}{10}$ b) $\frac{2}{25}$

c) $\frac{9}{50}$ d) $1 - \frac{7}{50} = \frac{43}{50}$

8) 1) 25% 2)12.5% 3)37.5%

9) a) $\frac{30}{40} = \frac{3}{4}$ b) $\frac{24}{40} = \frac{3}{5}$

c) $\frac{10}{40} = \frac{1}{4}$ d) $\frac{34}{40} = \frac{17}{20}$

10) First : $\frac{20}{42} = \frac{10}{21}$

Second : $\frac{14}{42} \times 600 = 200$

11) a) $\frac{11}{15}$ b) $\frac{4}{15}$

12) a) $\frac{56}{120} = \frac{7}{15}$ b) $\frac{40}{120} = \frac{1}{3}$ c) $\frac{88}{120} = \frac{11}{15}$

Third: Answer of pupil's book P. 137.

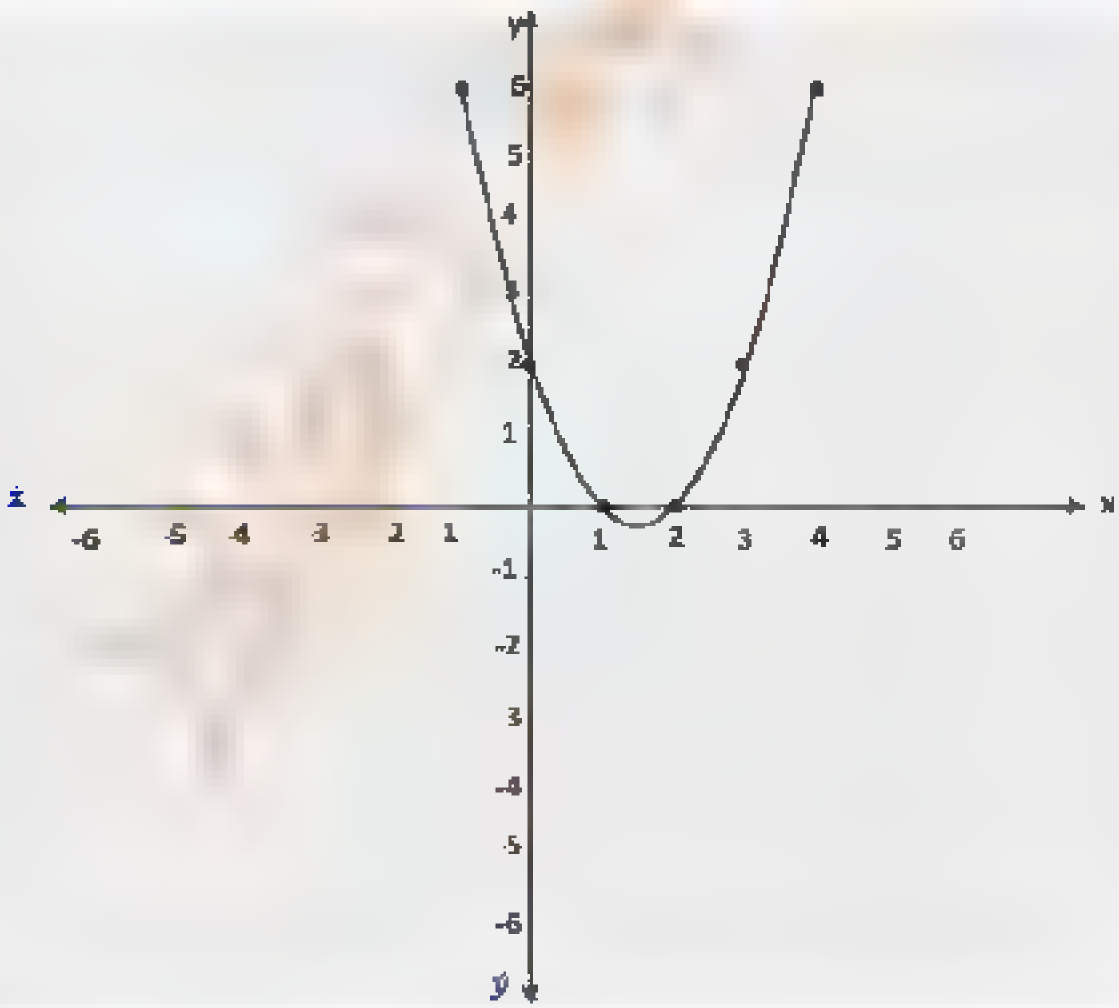
$f(x) = x^2 - 3x + 2$

X	-1	0	1	2	3	4
f(x)	6	2	0	0	2	6

vertex = (1.3 0.5)

min value = - 0.5

S.S. = {1 , 2}

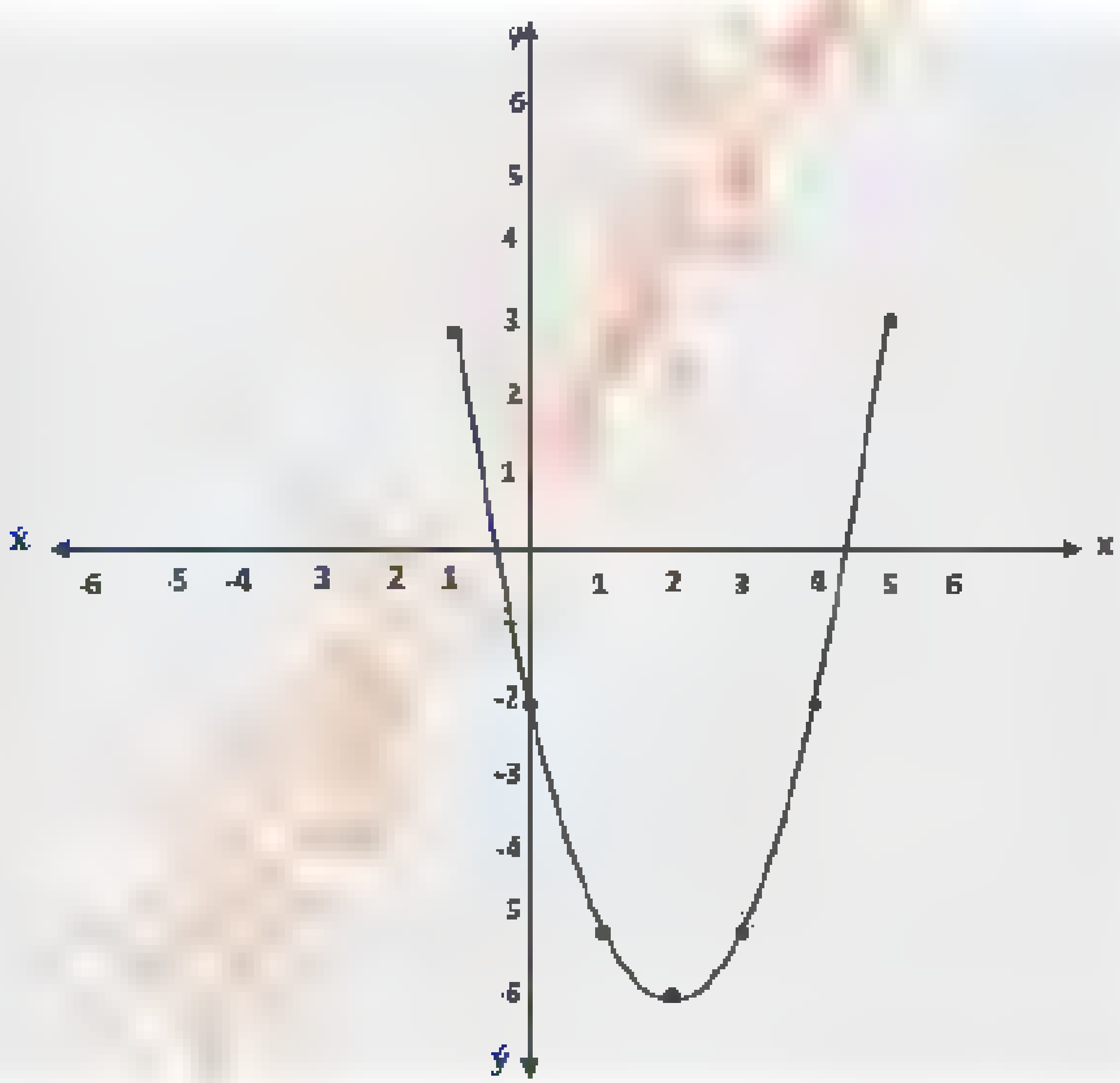


(2) $f(x) = x^2 - 4x - 2$

X	-1	0	1	2	3	4	5
f(x)	3	-2	-5	-6	-5	-2	3

min = - 6

S.S. = {-0.5 , 4.5}



1 EQUATIONS

Solving two equations in two variables – Solving two equations of second degree in one unknown – General formula – Set of zeroes of the polynomial functions.

SOLVING TWO EQUATION OF THE FIRST DEGREE IN TWO VARIABLES

1 The solution set of the two equation : $x + y = 0$, $y - 5 = 0$ in : $\mathbb{R} \times \mathbb{R}$ is (Alex 11 , Aswan 18)

ⓐ $\{5, -5\}$ ⓑ $\{(5, -5)\}$ ⓒ $\{(-5, 5)\}$ ⓓ $(-5, 5)$

2 The solution set of the two equation : $x - 2y = 1$, $3x + y = 10$ in : $\mathbb{R} \times \mathbb{R}$ is (Souhag 18 , Fayoum 11)

ⓐ $\{(5, 2)\}$ ⓑ $\{(2, 4)\}$ ⓒ $\{(1, 3)\}$ ⓓ $\{(3, 1)\}$

3 The two straighy lines : $x + 2y = 1$, $2x + 4y = 6$ are (South Sini 21 , Ismailia 21)

ⓐ Parallel ⓑ Perpendicular ⓒ Coincident ⓓ intersecting

4 The two straight lines : $3x + 5y = 0$, $5x - 3y = 0$ are intersected in the (Alex 14 , Beheira 11 , Assiut 21)

ⓐ Origin point ⓑ first quadrant ⓒ second quadrant ⓓ fourth quadrant

5 The Solution set of the two equations : $x = 3$, $y = 4$ is (Qalyubia 21 , Minya 21)

ⓐ $\{(3, 4)\}$ ⓑ $\{(4, 3)\}$ ⓒ $\mathbb{R}$ ⓓ ϕ

6 The S.S in $\mathbb{R} \times \mathbb{R}$ of the two equations : $y - 3 = 0$, $y + x = 0$ is (Ismailia 12)

ⓐ $\{3, 3\}$ ⓑ $\{(-3, 3)\}$ ⓒ $\{(3, 0)\}$ ⓓ $\{(0, 3)\}$

7 The two straight lines representing the two equations : $2x - y = 4$, $2x - 3 = y$ are

ⓐ Parallel ⓑ Perpendicular ⓒ Coincident ⓓ intersecting

8 The two straight lines representing the two equations : $x - y = 2$, $2x - 2y = 4$ are

ⓐ Parallel ⓑ Perpendicular ⓒ Coincident ⓓ intersecting

9 If there are infinite number of solutions in $\mathbb{R} \times \mathbb{R}$ of the two equations : $x + 4y = 7$, $3x + ky = 21$
Then : $k =$ .... (Souhag 19 , Beheira 18 , Qena 17 , Dakahlia 12 , Giza 21)

ⓐ 4 ⓑ 7 ⓒ 12 ⓓ 21

10 If the two equations : $x + 2y = 1$, $2x + ky = 2$ has only one solution for , Then $k \neq$ (Giza 18)

ⓐ 2 ⓑ 3 ⓒ 4 ⓓ -4

11 If the point of intersection of the two equations : $x - 3 = 0$ and $y + 2k = 5$ lies on the fourth quadrant , Then k may be equals

ⓐ -1 ⓑ -2 ⓒ 1 ⓓ 3

12 The number of solutions of the equation : $x + y = 5$ in $\mathbb{R} \times \mathbb{R}$ is

ⓐ zero ⓑ 1 ⓒ 2 ⓓ Infinite numbers

13 The number of solutions of the equation : $x = 3$ in $\mathbb{R}$ is

ⓐ zero ⓑ 1 ⓒ 2 ⓓ Infinite numbers

14 The number of solutions of the equation : $x = 7$ in $\mathbb{R} \times \mathbb{R}$ is ...... (Cairo 21)

ⓐ Infinite numbers ⓑ zero ⓒ 1 ⓓ 2

15 The point of intersection of the two straight lines : $x = 2$ and $x + y = 6$ is ...... (Alex 18)

ⓐ (2 , 6) ⓑ (2 , 4) ⓒ (4 , 2) ⓓ (6 , 2)

16 The point of intersection of the two straight lines : $2x - y = 3$ and $2x + y = 5$ lies on the ......

ⓐ First quadrant. ⓑ Second quadrant. ⓒ Third quadrant. ⓓ Fourth quadrant.

17 If the point of intersection of the two straight lines : $x - 1 = 0$ and $y = 2k$ lies on the fourth quadrant , then k may be equal ...... (K.El.Sheikh 16)

ⓐ -5 ⓑ 0 ⓒ 1 ⓓ 5

18 The two straight lines : $3x = 7$ and $2y = 9$ are ...... (matrouh 16 , luxor 17)

ⓐ Parallel ⓒ Coincide

ⓒ Intersect and non-perpendicular ⓓ perpendicular

19 If the two straight lines which represent the two equations : $x + 3y = 4$ and $x + a y = 7$ are parallel , then $a =$ ...... (Port Said 18)

ⓐ 6 ⓑ 1 ⓒ -3 ⓓ 3

20 If the point (9 , 2) belong to the set of solutions of the equation : $x - ky = 3$, then $k =$ ......

ⓐ 1 ⓑ 2 ⓒ 3 ⓓ 6

21 Two numbers their sum = 13 and their difference is 5 , then the two number are ......

ⓐ 7 and 6 ⓑ 8 and 5 ⓒ 9 and 4 ⓓ 10 and 3

22 Three years ago , ahmed's age was x years , then his age after 5 years is ......

ⓐ $x + 3$ ⓑ $x + 5$ ⓒ $x + 8$ ⓓ $x + 2$

23 A two-digit-number , ones digit is x and tens digit is y , then the number is ......

ⓐ $x + 10y$ ⓑ $y + 10x$ ⓒ xy ⓓ $x + y$

24 If the sum of ages of a father and his son now is 47 years , then the sum of their ages after 10 years = ... years (Giza 18)

ⓐ 27 ⓑ 37 ⓒ 57 ⓓ 67

25 If $(5 , x - 4) = (y + 2 , 3)$, then : $x + y =$ ...... (Luxor 18)

ⓐ 6 ⓑ 8 ⓒ 10 ⓓ 12

26 If $(5 , x + 1) = (y , 3)$, then : $x + y =$ ...... (Damietta 21)

ⓐ 3 ⓑ 5 ⓒ 7 ⓓ 9

27 The orderd pair which satisfies the equation : $x - y = 1$ is ...... (Red Sea 21)

ⓐ (1 , 1) ⓑ (2 , 1) ⓒ (1 , 2) ⓓ (0.5 , 1)

SOLVING AN EQUATION OF THE SECOND DEGREE IN ONE UNKNOWN

1 The solution set of the equation : $x^2+1=0$ in $\mathbb{R}$ is ...... (Beni Suef 18)

ⓐ $\{1\}$ ⓑ $\{-1,1\}$ ⓒ $\{-1\}$ ⓓ ϕ

2 The solution set of the equation : $x^2-4=0$ in $\mathbb{R}$ is ...... (South Sina 21 , matrouh 21)

ⓐ $\{-2,2\}$ ⓑ $\{-2\}$ ⓒ $\{2\}$ ⓓ ϕ

3 If the curve of the quadratic function f passes through the points $(-1,0)$, $(0,-4)$, $(4,0)$ and And $(0,-6)$, Then the solution set of the equation : $f(x)=0$ in $\mathbb{R}$ is ...... (gharbia 19)

ⓐ $\{-1,0\}$ ⓑ $\{-4,0\}$ ⓒ $\{-1,4\}$ ⓓ $\{4,-4\}$

4 If the curve of the quadratic function f does not intersect X-axis at any points. , Then the number of solution of the equation : $f(x)=0$ in $\mathbb{R}$ is ...... (monofia 17)

ⓐ A unique solution ⓑ zero ⓒ two solution ⓓ An infinite solutions

5 The curve of the function f such that $f(x)=x^2-3x+2$ cuts X – axis at the two points ......

ⓐ $(2,0),(3,0)$ ⓑ $(2,0),(1,0)$ ⓒ $(-2,0),(-1,0)$ ⓓ $(2,0),(-1,0)$

6 The solution set of the equations : $x^2+5x=0$ in $\mathbb{R}$ is ......

ⓐ $\{0,5\}$ ⓑ $\{\frac{-5}{2},0\}$ ⓒ $\{2,5\}$ ⓓ ϕ

7 The solution set of the equations : $x^2-4x+4=0$ in $\mathbb{R}$ is ......

ⓐ $\{-2,2\}$ ⓑ $\{4,1\}$ ⓒ $\{2\}$ ⓓ ϕ

8 The solution set of the equations : $x^2+5=0$ in $\mathbb{R}$ is ......

ⓐ $\{\sqrt{5},-\sqrt{5}\}$ ⓑ $\{-\sqrt{5}\}$ ⓒ $\{\sqrt{5}\}$ ⓓ ϕ

9 In the equations : $ax^2+bx+c=0$ if $b^2-4ac>0$, then the equation has ...... roots in $\mathbb{R}$ (Fayoum 19 , damietta 18)

ⓐ 1 ⓑ 2 ⓒ zero ⓓ An infinite solutions

10 If the curve of the function $f : f(x)=ax^2+bx+c$ has a minimum value at $x=2$, then the number of solutions of the equation : $f(x)=0$ in $\mathbb{R}$ is ......

ⓐ 0 ⓑ 1 ⓒ 2 ⓓ An infinite solutions

11 If the point of the vertex of the curve of the function $f : f(x)=x^2+bx+c$ is $(2,8)$. , then the solution set of the equation : $f(x)=0$ in $\mathbb{R}$ is ......

ⓐ $\{2,6\}$ ⓑ $\{-2,6\}$ ⓒ $\{2,-6\}$ ⓓ $\{2,0\}$

12 If the equation of the symmetry line of the curve of the function $f : f(x)=x^2+bx-10$ is $x=\frac{3}{2}$. , then the solution set of the equation : $f(x)=0$ in $\mathbb{R}$ is ......

ⓐ $\{10,-1\}$ ⓑ $\{-2,5\}$ ⓒ $\{2,-5\}$ ⓓ $\{-1,10\}$

13 If : $x = 3$ is one of the solutions of the function $f : f(x) = x^2 - a x + 3$ in $\mathbb{R}$, Then : $a =$ ...... (Suez 17)

ⓐ 2 ⓑ 1 ⓒ −1 ⓓ −8

14 If the solution set of the equation : $x^2 - a x + 4 = 0$ in $\mathbb{R}$ is $\{-2\}$, Then : $a =$ ...... (Gharbia 18)

ⓐ −2 ⓑ −4 ⓒ 4 ⓓ 2

15 If the curve of the function $f : f(x) = x^2 - x + c$ passes through the points $(2 , 1)$, then : $c =$ ...... (gharbia 18)

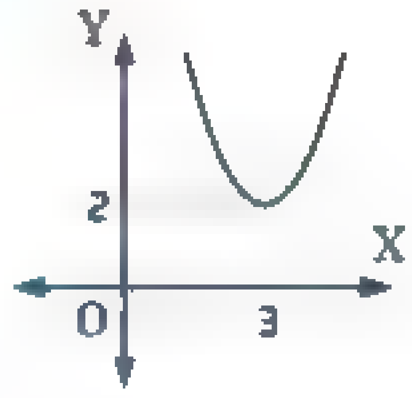
ⓐ 2 ⓑ 1 ⓒ −2 ⓓ −1

16 The solution set of the equation : $a x^2 + b x + c = 0 , a \neq 0$ graphically is the set of X coordinates of the point of intersection of the curve of the function $f : f(x) = a x^2 + b x + c$ with the ...... (Damietta 18)

ⓐ Y − axis ⓑ X − axis ⓒ Symmetric line ⓓ Straight line $y = 2$

17 **In the opposite figure :**

The solution set of $f : f(x) = 0$ is ...... (Souhag 18)



ⓐ ϕ ⓑ $\{3\}$

ⓒ $\{2 , 3\}$ ⓓ $\{2\}$

18 The equation of the symmetric axis of the curve of the function f where $f(x) = x^2 - 4$ is ...... (K.El.Sheikh 21)

ⓐ $x = -4$ ⓑ $x = 0$ ⓒ $y = 0$ ⓓ $y = -4$

SOLVING TWO EQUATION OF THE FIRST DEGREE IN TWO VARIABLES

1 The S.S of the two equations : $x - y = 0$, $x y = 9$ in $\mathbb{R} \times \mathbb{R}$ is ...... (Qena 18 , gharbia 11)

ⓐ $\{(0 , 0)\}$ ⓑ $\{(-3 , 3)\}$ ⓒ $\{(3 , 3)\}$ ⓓ $\{(3 , 3) , (-3 , -3)\}$

2 If : $x + y = 3$, $x^2 - y^2 = 6$ in $\mathbb{R} \times \mathbb{R}$, then $(x - y) =$ ......

ⓐ 18 ⓑ 9 ⓒ 3 ⓓ 2

3 If : $x^2 + y^2 = 9$, $(x + y)^2 = 17$, then : $x y =$ ......

ⓐ 16 ⓑ 8 ⓒ 4 ⓓ 2

4 one of the solutions of the two equations : $x - y = 2$, $x^2 + y^2 = 20$ is ...... (Kalyoubia 19 , Qena 17)

ⓐ $(-4 , 2)$ ⓑ $(2 , -4)$ ⓒ $(3 , 1)$ ⓓ $(4 , 2)$

5 The degree of the equation : $3x + 4y + xy = 5$ is ...... (Port Said 18 , Beheira 21)

ⓐ Zero ⓑ First ⓒ Second ⓓ Third

6 one solution of the equation : $x^2 - y^2 = 3$ in $\mathbb{R} \times \mathbb{R}$ is ......

ⓐ $(1 , -2)$ ⓑ $(-2 , 1)$ ⓒ $(1 , 2)$ ⓓ $(-1 , -2)$

7 The ordered pair that satisfies each of the two equations : $xy = 2$, $x - y = 1$ is ...... (Sharkia 12)

ⓐ $(1,1)$ ⓑ $(2,1)$ ⓒ $(1,2)$ ⓓ $(2,-1)$

8 The solution set of the two equations : $x = y$, $xy = 1$ in $\mathbb{R} \times \mathbb{R}$ is ......

ⓐ $\{(1,1)\}$ ⓑ $\{(-1,-1)\}$ ⓒ $\{(1,-1)\}$ ⓓ $\{(1,1),(-1,-1)\}$

9 If : $y = 1 - x$, $(x + y)^2 + y = 5$, Then : $y =$ ...... (Fayoum 12)

ⓐ 5 ⓑ 3 ⓒ 4 ⓓ -4

10 If : $x^2 + xy = 15$, $x + y = 5$, Then : $x =$ ......

ⓐ 3 ⓑ 4 ⓒ 5 ⓓ 6

11 If the difference between two numbers is 1 and the square of their sum is 25 , then the two numbers are ......

ⓐ 1 , 2 ⓑ 2 , 3 ⓒ 3 , 4 ⓓ 4 , 5

12 Two positive numbers , their sum is 9 and their product is 8 , then the two numbers are ...... (Giza 12)

ⓐ 2 , 7 ⓑ 3 , 6 ⓒ 4 , 9 ⓓ 1 , 8

13 If : $x^2y + xy^2 = 25$, $x + y = 5$, Then : $xy =$ ......

ⓐ 3 ⓑ 4 ⓒ 5 ⓓ 6

14 If : $x + 2y = 5$, $(x + 2y - 3)^2 + 2x = 10$, Then : $x =$ ......

ⓐ 2 ⓑ 3 ⓒ 6 ⓓ -3

SET OF ZEROES OF A POLYNOMIAL FUNCTIONS

1 The set of zeroes of the function $f : f(x) = -3x$ is ...... (Alex 12 , Giza 17 , Suez 18 , Damietta 21)

ⓐ $\{0\}$ ⓑ $\{-3\}$ ⓒ $\{-3,0\}$ ⓓ $\mathbb{R}$

2 The set of zeroes of the function f where $f(x) = 4$ is ...... (Aswan 12 , Aswan 17 , Matrouh 19 , Minya 21)

ⓐ $\{-4\}$ ⓑ $\{0\}$ ⓒ ϕ ⓓ $\{2\}$

3 The set of zeroes of the function f where $f(x) =$ zero is ...... (Cairo 19 , Ismailia 21)

ⓐ ϕ ⓑ $\mathbb{R} - \{0\}$ ⓒ $\mathbb{R}$ ⓓ zero

4 The set of zeroes of the function f where $f(x) = x^2 + 9$ is ...... (Dakahlia 19)

ⓐ $\mathbb{R}$ ⓑ $\{3\}$ ⓒ $\{3,-3\}$ ⓓ ϕ

5 The set of zeroes of the function f where $f(x) = x^2 - 4$ is ...... (Qalyubia 21)

ⓐ $\{2\}$ ⓑ $\{2,-2\}$ ⓒ $\mathbb{R}$ ⓓ ϕ

6 The set of zeroes of the function f where $f(x) = x(x^2 - 2x + 1)$ is ...... (Alex 13 , Ismailia 17)

ⓐ $\{0,1\}$ ⓑ $\{0,-1\}$ ⓒ $\{-1,1\}$ ⓓ $\{0,1,-1\}$

7 The set of zeroes of f where $f(x) = x - 5$ is ...... (Dumietta 11 Suez 19 , Matrouh 21)

ⓐ $\{zero\}$ ⓑ $\{5\}$ ⓒ $\{-5\}$ ⓓ $\{-5, 5\}$

8 The set of zeroes of f where $f(x) = (x-1)^2 (x+2)$ is ...... (Suez 12)

ⓐ $\{1, -2\}$ ⓑ $\{-1, 2\}$ ⓒ $\{-1, -2\}$ ⓓ $\{1, 2\}$

9 If : $z(f) = \{2\}$, $f(x) = x^3 - m$, then : $m =$ ...... (Ismailia 12 , Sharkia 14 , Qena 15)

ⓐ $\sqrt[3]{2}$ ⓑ 2 ⓒ 4 ⓓ 8

10 If : $z(f) = \{1, -2\}$, $f(x) = x^2 + x + a$, then : $a =$ ...... (Sharkia 14 , Qena 15)

ⓐ 28 ⓑ 1 ⓒ -1 ⓓ -2

11 If : $z(f) = \{5\}$, $f(x) = x^3 - 3x^2 + a$, then : $a =$ ...... (Assiut 11 , Port said 14)

ⓐ $\sqrt[3]{2}$ ⓑ 2 ⓒ 4 ⓓ 8

12 If $\{-2, 2\}$ is the set of zeroes of the function f where $f(x) = x^2 + a$, then : $a =$ ...... (Sharkia 21)

ⓐ 2 ⓑ -2 ⓒ 4 ⓓ -4

13 If the set of zeroes of the function f : $f(x) = x^2 + kx + 1$ is ϕ , then : k may equal ...... (Sharkia 15)

ⓐ 3 ⓑ 2 ⓒ 1 ⓓ -2

14 If the set of zeroes of the function f where $f(x) = ax + (b-2)$ is $\{2\}$, $a - b = 4$, then : $a =$ ......

ⓐ 1 ⓑ 2 ⓒ -2 ⓓ -1

15 If the set of zeroes of the function f : $f(x) = x^2 + ax + 4$ equals to the set of zeroes of the Function $g : g(x) = x - 2$, then : $a =$ ......

ⓐ 2 ⓑ -2 ⓒ -4 ⓓ 4

16 If $\{3\}$ is the set of common zeroes between the two function f : $f(x) = x^2 - ax$ and $g : g(x) = ax + b$, then : $b =$ ......

ⓐ 3 ⓑ -3 ⓒ 9 ⓓ -9

17 If : $a \in$ the set of zeroes of the function $f(x) = x^2 - 2x - 3$ and $a \notin$ the set of zeroes of the function $g(x) = x + 1$, then : $a \in$ ......

ⓐ $\{3\}$ ⓑ $\{-1, 3\}$ ⓒ $\{-1, 3, 5\}$ ⓓ $\{3, 5\}$

18 If : $f(x) = ax^2 + bx + c$ and $f(x) = 0$ to each $x \in \{0, 2\}$, then : $2a + b + c =$ ......

ⓐ 2 ⓑ 4 ⓒ 0 ⓓ 20

19 The set of zeroes of the function f where $f(x) = x^3 - 3x^2 - 4x + 12$ is ......

ⓐ $\{3\}$ ⓑ $\{-2, 2\}$ ⓒ $\{-2, 2, 3\}$ ⓓ $\{2, 3\}$

2 ALGEBRIC FRACTIONS

The domain of the algebraic fractions – The common domain

Reducing the algebraic fractions – Operations on the algebraic fractions.

ALGEBRAIC FRACTIONAL FUNCTION

1 The domain of the function f where $f(x) = \frac{x+2}{x-1}$ is ......

ⓐ $\{1\}$ ⓑ $\{-2\}$ ⓒ $\mathbb{R}-\{2\}$ ⓓ $\mathbb{R}-\{1\}$

2 The domain of the function f where $f(x) = \frac{x^2-x}{x^2-2x-3}$ is ......

ⓐ $\mathbb{R}-\{0\}$ ⓑ $\mathbb{R}-\{-1,3\}$ ⓒ $\mathbb{R}-\{0,1\}$ ⓓ $\mathbb{R}-\{1,-3\}$

3 The domain of the function f where $f(x) = \frac{x+2}{5x}$ is ...... (Kalyoubia 17)

ⓐ $\mathbb{R}-\{5\}$ ⓑ $\mathbb{R}-\{-5\}$ ⓒ $\mathbb{R}$ ⓓ $\mathbb{R}-\{\text{zero}\}$

4 The domain of the function f where $f(x) = \frac{x^2+2}{x^2+4}$ is ......

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{-2,2\}$ ⓒ $\{-2,2\}$ ⓓ $\{2,3\}$

5 The domain of the function f where $f(x) = \frac{x(x+2)}{x^2-4}$ is ......

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{-2,2\}$ ⓒ $\mathbb{R}-\{0,2\}$ ⓓ $\mathbb{R}-\{2\}$

6 The domain of the function f where $f(x) = \frac{x-3}{2}$ is ...... (Giza 17)

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{0\}$ ⓒ $\mathbb{R}-\{-1,0\}$ ⓓ $\mathbb{R}-\{1,0\}$

7 The domain of the function f where $f(x) = \frac{x-7}{3(x+1)}$ is ......

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{1\}$ ⓒ $\mathbb{R}-\{-1,3\}$ ⓓ $\mathbb{R}-\{-1\}$

8 If : $f(x) = \frac{x^2-9}{x+b}$, $f(4) = 1$, then : $b =$ ......

ⓐ -7 ⓑ 7 ⓒ 3 ⓓ -3

9 If : $n(x) = \frac{7}{x+a}$, and the domain of the function n is $\mathbb{R}-\{-2\}$, then : $a =$ ...... (Monofia 11)

ⓐ -2 ⓑ 2 ⓒ 0 ⓓ 7

10 The domain of the function f where $f(x) = x^2-4$ is ...... (Dakahlia 21)

ⓐ $\mathbb{R}-\{2,-2\}$ ⓑ $\mathbb{R}-\{0\}$ ⓒ $\mathbb{R}$ ⓓ ϕ

11 If : $n_1(x) = \frac{-7}{x+2}$, $n_2(x) = \frac{x}{x-k}$ and The common domain of the two functions n_1 and n_2 is $\mathbb{R}-\{-2,7\}$, then $k =$ ...... (North Sinai 12)

ⓐ 2 ⓑ -2 ⓒ 7 ⓓ -7

12 The domain of the function f : $f(x) = \frac{x^2 - 5x - 14}{x^2 + 9}$ is ...... (Arab republic of Egypt 21)

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R} - \{-3\}$ ⓒ $\mathbb{R} - \{3, -3\}$ ⓓ $\mathbb{R} - \{2, -7\}$

13 If : $n_1(x) = \frac{x+2}{x-1}$, $n_2(x) = \frac{x-5}{x+3}$, then The common domain of the two functions is ...... (Cairo 12)

ⓐ $\mathbb{R} - \{1, -2\}$ ⓑ $\mathbb{R}$ ⓒ $\mathbb{R} - \{5, -3\}$ ⓓ $\mathbb{R} - \{1, -3\}$

14 The common domain of the two functions n_1 and n_2 where : $n_1(x) = 3x - 15$, $n_2(x) = x^2 - 4$ is ......

ⓐ $\mathbb{R} - \{5\}$ ⓑ $\mathbb{R} - \{-3\}$ ⓒ $\mathbb{R} - \{2, -2, 5\}$ ⓓ $\mathbb{R}$

15 The common domain of the two functions : $f_1(x) = \frac{1}{x-1}$, $f_2(x) = \frac{1}{x^2+4}$ is ...... (Sharkia 12)

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R} - \{1\}$ ⓒ $\mathbb{R} - \{1, 2\}$ ⓓ $\mathbb{R} - \{1, 2, -2\}$

16 If the domain of the algebraic fraction n is $\mathbb{R} - \{2, 3, 4\}$, then : $n(3) =$ ...... (Sharkia 19)

ⓐ 3 ⓑ 2 ⓒ 4 ⓓ Undefined

17 If the domain of the function n : $n(x) = \frac{x+2}{4x^2 + kx + 9}$ is $\mathbb{R} - \{\frac{-3}{2}\}$, then : $k =$ ...... (Kafr El Sheikh 19)

ⓐ 3 ⓑ 2 ⓒ 4 ⓓ Undefined

THE SET OF ZEROES OF THE ALGEBRAIC FRACTIONAL FUNCTIONS

1 The set of zeroes of the function f where $f(x) = \frac{x^2 - 9}{x - 3}$ is ...... (Monofia 17)

ⓐ $\{3\}$ ⓑ $\{-3\}$ ⓒ $\{-3, 3\}$ ⓓ ϕ

2 The set of zeroes of the function f where $f(x) = \frac{x^2 - x - 2}{x^2 + 4}$ is ...... (Gharbia 17)

ⓐ $\{2, -2\}$ ⓑ $\{-2, -1\}$ ⓒ $\{2, -1\}$ ⓓ $\{1, -1\}$

3 The set of zeroes of the function f where $f(x) = \frac{(x-5)(x-4)}{x^2 + 16}$ is ...... (monofia 12)

ⓐ $\{2, -2\}$ ⓑ $\{-2, -1\}$ ⓒ $\{2, -1\}$ ⓓ $\{1, -1\}$

4 The set of zeroes of the function f where $f(x) = \frac{x^3 + 2x^2 - 4x - 8}{x^2 - 4}$ is ......

ⓐ $\mathbb{R}$ ⓑ $\{-2, 2\}$ ⓒ $\mathbb{R} - \{-2, 2\}$ ⓓ ϕ

5 If the set of zeroes of the function f where $f(x) = \frac{x^{\blacksquare} + 4x - 12}{x + k}$ is $\{-6\}$ then : $k =$ ......

ⓐ 2 ⓑ -2 ⓒ 1 ⓓ 6

6 If the set of zeroes of the function f where $f(x) = \frac{x^2 - k}{x + 2}$ is $\{2\}$ then : $k =$ ......

ⓐ 2 ⓑ -2 ⓒ 4 ⓓ -4

REDUCING THE ALGEBRAIC FRACTIONS

1 The Simplest form of the function f where $f(x) = \frac{2x^2+x}{x}$ and $x \neq 0$ is ...... (Giza 12)

ⓐ $3x$ ⓑ $2x^2+1$ ⓒ x^2+1 ⓓ $2x+1$

2 The Simplest form of the function $f : f(x) = \frac{5-x}{x-5}$ and $x \neq 5$ is ...... (Sharkia 12)

ⓐ 5 ⓑ 0 ⓒ -1 ⓓ 1

3 The Simplest form of the function $f : f(x) = \frac{x}{x-1} + \frac{1}{1-x}$ and $x \neq 1$ is ......

ⓐ $\frac{x+1}{x-1}$ ⓑ $\frac{x+1}{1-x}$ ⓒ 1 ⓓ -1

4 The simplest form of the function $n : n(x) = \frac{3-x}{x-3}$ such that $x \in \mathbb{R} - \{3\}$ is ...... (Dakahlia 17)

ⓐ 1 ⓑ -1 ⓒ 3 ⓓ -3

5 The simplest form of : $n(x) = \frac{x^2+1}{x^2+4} + \frac{3}{x^2+4}$ is ...... (Fayoum 15)

ⓐ 3 ⓑ 4 ⓒ 1 ⓓ $\frac{1}{x^2+4}$

OPERATIONS ON THE ALGEBRAIC FRACTIONS

1 The additive inverse of the fraction $n : n(x) = \frac{x-1}{x+3}$ and $x \neq -3$ is ......

ⓐ $\frac{x+1}{x-3}$ ⓑ $\frac{1-x}{x+3}$ ⓒ $\frac{x+1}{-(x+3)}$ ⓓ $\frac{1-x}{-(x+3)}$

2 The fraction f where $f(x) = \frac{x-2}{x-5}$ has an additive inverse if the domain is ......

ⓐ $\mathbb{R} - \{2\}$ ⓑ $\mathbb{R} - \{5\}$ ⓒ $\mathbb{R} - \{-2, 2\}$ ⓓ $\mathbb{R} - \{0, 1\}$

3 If : $n(x) = \frac{x-2}{x+5}$, then the domain of n^{-1} is ...... (Gharbia 17 , Sharkia 18 , Port Said 19)

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R} - \{2\}$ ⓒ $\mathbb{R} - \{-5\}$ ⓓ $\mathbb{R} - \{2, -5\}$

4 If : $n(x) = \frac{x}{x^2+1}$, then the domain of n^{-1} is ...... (Sharkia 21)

ⓐ $\mathbb{R} - \{0\}$ ⓑ ϕ ⓒ $\mathbb{R} - \{-1\}$ ⓓ $\mathbb{R} - \{1, -1\}$

5 The multiplicative inverse of the fraction : $n(x) = \frac{x-3}{x^2-9} \times \frac{x-3}{x}$ is ......

ⓐ $\frac{1}{x}$ ⓑ $-\frac{1}{x}$ ⓒ x ⓓ $-x$

6 The fraction f where $f(x) = \frac{x-2}{x-5}$ has a multiplicative inverse if the domain is ......

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R} - \{5\}$ ⓒ $\mathbb{R} - \{2\}$ ⓓ $\mathbb{R} - \{2, 5\}$

7 If : $n(x) = \frac{x-3}{x^2-4}$, then : $n^{-1}(3) =$ ......

ⓐ ϕ ⓑ $\mathbb{R} - \{3, -3\}$ ⓒ $\mathbb{R} - \{0\}$ ⓓ $\mathbb{R}$

8 If : $(x \neq 0)$, then : $n(x) = \frac{5x}{x^2+1} \div \frac{x}{x^2+1} =$ ------ (Souhag 19 , S-Sini 21)

ⓐ -5 ⓑ -1 ⓒ 5 ⓓ 1

9 If : $n(x) = \frac{x-2}{x+1}$, then : $n^{-1}(2) =$ ------ (Dakahlia 21)

ⓐ 0 ⓑ 2 ⓒ 3 ⓓ undefined

10 The domain of the multiplicative inverse of the function : $n(x) = \frac{x+2}{x-3}$ is ------ (Beheira 21)

ⓐ $\mathbb{R}-\{3\}$ ⓑ $\mathbb{R}-\{-3\}$ ⓒ $\mathbb{R}-\{-2,3\}$ ⓓ $\mathbb{R}$

11 The domain of the Additive inverse of the function : $n(x) = \frac{x-2}{x-5}$ is ------ (Port Said 21)

ⓐ $\mathbb{R}-\{2,5\}$ ⓑ $\mathbb{R}-\{2\}$ ⓒ $\mathbb{R}-\{5\}$ ⓓ $\{2,5\}$

12 If the algebraic fraction $\frac{x-a}{x-2}$ has a multiplicative inverse which is $\frac{x-2}{x+3}$, then : $a =$ ------ (Beni Suef 21)

ⓐ $\mathbb{R}-\{2,5\}$ ⓑ $\mathbb{R}-\{2\}$ ⓒ $\mathbb{R}-\{5\}$ ⓓ $\{2,5\}$

13 If : $n(x) = \frac{x^2-2x}{(x-3)(x^2+2)}$, then the domain of n^{-1} is ------ (Fayoum 19)

ⓐ $\mathbb{R}$ ⓑ $\mathbb{R}-\{2\}$ ⓒ $\mathbb{R}-\{0\}$ ⓓ $\mathbb{R}-\{0,2\}$

14 If the domain of the function : $n(x) = \frac{1}{x} + \frac{9}{x+b}$ is $\mathbb{R}-\{0,4\}$, then : $b =$ ------ (Giza 12)

ⓐ 0 ⓑ -4 ⓒ 4 ⓓ 3

15 The function f where $f(x) = \frac{x-2}{x^3+27}$, then the domain of its multiplicative inverse is ------ (Port Said 17)

ⓐ $\mathbb{R}-\{2\}$ ⓑ $\mathbb{R}-\{-3,2\}$ ⓒ $\mathbb{R}-\{2,-3,3\}$ ⓓ $\mathbb{R}-\{-3,3\}$

3 PROBABILITY

Operation on the events

(union – intersection – difference and complement)

1 The probability of the impossible event equals ......

(Kafr El Sheikh 17 , Beni Suef 17 , South Sini 19 , Cairo 21 , Kalyoubia 21)

ⓐ ϕ ⓑ Zero ⓒ $\frac{1}{2}$ ⓓ 1

2 If : A and B are two mutually exclusive events , then : $P (A \cap B) =$ ......

(Giza 11 , Cairo 12 , Gharbia 15 , Monofia 17 , Fayoum 17 , Cairo 19 , Ismailia 19 , Red Sea 21 , Dakahlia 21)

ⓐ ϕ ⓑ P (A) ⓒ P (B) ⓓ Zero

3 If : A and B are two events in a sample space for a random experiments , $A \subset B$, then : $P (A \cap B) =$ ...... (Kalyoubia 12 , Cairo 16)

ⓐ P (B) ⓑ P (A) ⓒ Zero ⓓ ϕ

4 If : $A \subset B$, then : $P (A \cup B) =$ ...... (Gharbia 12 , Qena 17 , Kalyoubia 18 , Beheira 19 , Aswan 19 , Dakahlia 19)

ⓐ Zero ⓑ P (A) ⓒ P (B) ⓓ $P (A \cap B)$

5 If a regular coin is tossed once , then the probability of getting head or tail is ...... (Dakahlia 13 , Alex 14)

ⓐ 100 % ⓑ 50 % ⓒ 25 % ⓓ zero

6 If a regular die is rolled once , then the probability of getting an odd number and even number together equals ...... (Fayoum 12 , Beheira 14 , Alex 16)

ⓐ Zero ⓑ $\frac{1}{2}$ ⓒ $\frac{3}{4}$ ⓓ 1

7 If a regular die is rolled once , then the probability of getting an odd number and a prime number together equals ...... (Port Said 19 , Kafr El Sheikh 21)

ⓐ $\frac{1}{6}$ ⓑ Zero ⓒ $\frac{3}{4}$ ⓓ 1

8 If a regular coin is tossed once , then the probability of getting tail is ...... (Beni Suef 19)

ⓐ $\frac{1}{4}$ ⓑ $\frac{1}{2}$ ⓒ $\frac{3}{4}$ ⓓ 1

9 If : A and B are two mutually exclusive events , P (B) = 0.5 and $P (A \cup B) = 0.7$, then : P (A) = ... (Alex 17)

ⓐ 0.02 ⓑ 0.2 ⓒ 0.5 ⓓ 0.13

10 If the probability that a student succeeded is 95 % , then the probability that he does not succeed is ...... (Aswan 17)

ⓐ 20 % ⓑ 5 % ⓒ 5010 % ⓓ zero

11 If : A and B are two mutually exclusive events , then : $A \cap B =$ ...... (Assiut 21 , Alex 21 , Cairo 21)

ⓐ zero ⓑ 0.5 ⓒ 1 ⓓ ϕ

12 If : A and B are two events in a sample space , then the event of occurrence of A only is = ... (Menia 15)

ⓐ $A`$ ⓑ $A - B$ ⓒ $A \cap B$ ⓓ $A \cup B$

13 If : A is an event from the sample space of a random experiment , $P(A`) =$ ...... (Dakahlia 17)

ⓐ 1 ⓑ -1 ⓒ $1 - P(A)$ ⓓ $P(A) - 1$

14 If : $P(A) = 4P(A`)$, then : $P(A) =$ ...... (Kalyoubia 17 , Kalyoubia 18)

ⓐ 0.8 ⓑ 0.6 ⓒ 0.4 ⓓ 0.2

15 If : $P(A) = \frac{1}{3}$, then : $P(A`) =$ ...... (Giza 12 , Assiut 17)

ⓐ $\frac{1}{3}$ ⓑ $\frac{2}{3}$ ⓒ 1 ⓓ $\frac{1}{2}$

16 If : $P(A) = P(A`)$, then : $P(A) =$ ...... (Alex 12 , Dakahlia 12 , Giza 17 , Suez 19)

ⓐ Zero ⓑ 1 ⓒ $\frac{1}{2}$ ⓓ $\frac{1}{3}$

17 If : $X \subset S$ and $X`$ is the complementary event to event X , then : $P(X \cap X`) =$ ...... (Assiut 19)

ⓐ zero ⓑ S ⓒ ϕ ⓓ 1

18 If : A and B are two events in a sample space of a random experiment , $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$

$P(A \cup B) = \frac{5}{6}$, then : $P(A \cap B) =$ ...... (Dakahlia 12)

ⓐ $\frac{1}{2}$ ⓑ $\frac{1}{3}$ ⓒ $\frac{1}{5}$ ⓓ $\frac{1}{6}$

19 If : A and B are two events in a random experiment and $A \subset B$, then : $P(A - B) =$ ......

ⓐ zero ⓑ $P(A) - P(B)$ ⓒ $P(B) - P(A)$ ⓓ $P(A)$

20 If : $A \subset S$ of a random experiment and $P(A`) = 2P(A)$, then : $P(A) =$ ...... (Alex 19 , Port Said 19)

ⓐ $\frac{1}{3}$ ⓑ $\frac{1}{2}$ ⓒ $\frac{2}{3}$ ⓓ 1

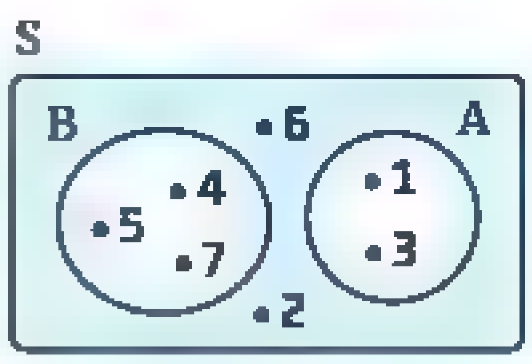
21 If : $P(A) + P(A`) = 2k$, then : $k =$ ...... (Giza 19)

ⓐ 1 ⓑ $\frac{1}{2}$ ⓒ $\frac{1}{3}$ ⓓ $\frac{1}{4}$

22 If : $A \cap B = \phi$, then : $P(A - B) =$ ...... (Kalyoubia 19)

ⓐ $P(A)$ ⓑ $P(B)$ ⓒ $P(B - A)$ ⓓ 1

23 **In The opposite Figure :**



If : A and B are two events in a sample space of a random experiment

Then , $P(B - A) =$ ...... (Kafr El Sheikh 19)

ⓐ $\frac{1}{2}$ ⓑ $\frac{5}{7}$ ⓒ $\frac{2}{7}$ ⓓ $\frac{3}{7}$

1 EQUATIONS

Solving two equations in two variables - Solving two equations of second degree in one unknown - General formula - Set of zeroes of the polynomial functions.

SOLVING TWO EQUATION OF THE FIRST DEGREE IN TWO VARIABLES

1 Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations

[1] $x + y = 5$ and $x - y = 1$ (S-Sini 13 , Kafr El Sheikh 21) $\{(3,2)\}$

[2] $2x - y = 3$ and $x + 2y = 4$ (N-Valley 12 , Alex 18 , Sharkia 19 , Damietta 21) $\{(2,-1)\}$

[3] $y = x + 4$ and $x + y = 4$ (Souhag 16 , Gharbia 19 , S-Sini 21) $\{(0,4)\}$

[4] $3x + 4y = 24$ and $x - 2y = -2$ (Giza 12 , Gharbia 18) $\{(4,3)\}$

[5] $2x + 3y = 7$ and $3x + 2y = 8$ $\{(2,1)\}$

2 **Find in $\mathbb{R} \times \mathbb{R}$ the S.S of the two Equations :** $\frac{x}{6} + \frac{y}{3} = \frac{1}{3}$ and $\frac{x}{2} + \frac{2y}{3} = 1$ $\{(2, 0)\}$

3 **Find the values of** a and b knowing that $(3, -1)$ is the solution set of the two equations :

$a x + b y - 5 = 0$ and $3 a x + b y = 17$ (Gharbia 16 , Damietta 17 , Luxor 18) ($a = 2, b = 1$)

4 **Find the values of** a and b , If : $(a, 2a)$ is a solution for the two equations :

$3x - y = 5$ and $x + y = -1$ (Dakahlia 17) ($a = 1, b = -1$)

5 A rectangle is with a length more than its width by 4 cm. If the perimeter of rectangle is 28 cm.

Find the area of the rectangle. (Alex 12 , Cairo 17 , Kalyoubia 19) (45 cm^2)

6 Two acute angles in a right-angled triangle , the difference between their measures is 50°.

Find the measure of each angle. (Sharkia 12 , Damietta 17 , Kalyoubia 18 , Beheira 19 , Dakahlia 21) (70° , 20°)

7 A two-digit number , the sum of its digits is 11 , if the two digits reversed , then the resulted number will be more than the original number by 9. **what is the original number.** (Kafr El Sheikh 16) (45)

8 **Find algebraically in $\mathbb{R} \times \mathbb{R}$ the solution set of :**

the equations represented by the two straight lines : $2x - y = 1$ and the straight line passes through the two points $(1, -2)$ and $(-1, -4)$ $\{(-2,5)\}$

9 If the straight lines whose equations are : $x + y = 3$, $3x - 2y + 1 = 0$ and $y + kx = 4$ intersects at the same point , then **Find the value of** k . ($k = 2$)

10 If the two points $(3, 1)$, $(5, 5)$ lies on the straight line $a x + b y = 5$

Find the value of a and b. ($a = 2, b = -1$)

SOLVING AN EQUATION OF THE SECOND DEGREE IN ONE UNKNOWN

1 **Find in $\mathbb{R}$ the solution set of the following equations using the general formula**

1. $x^2 - 4x + 1 = 0$ (rounding the result to two decimal digits) (Beheira 11 , Alex 13 , Aswan 14 , Giza 17) $\{0.27 , 3.73\}$

2. $2x^2 - 4x + 1 = 0$ (Approximating the result to the nearest three decimal places) (Qena 12 , Dakahlia 19 , Kalyoubia 19) $\{0.293 , 1.707\}$

3. $3x^2 = 5x - 1$ (Approximating the result to two decimals) (Helwan 11 , Luxor 17 , Monofia 19) $\{0.23 , 1.43\}$

4. $x(x - 1) = 4$ (taking $\sqrt{17} \approx 4.12$) (Sharkia 17 , Souhag 19) $\{-1.56 , 2.56\}$

5. $x + \frac{4}{x} = 6$ (rounding the result to one decimal digit) (Dumietta 19) $\{0.8 , 5.2\}$

6. $(x - 4)(x - 2) = 1$ (taking $\sqrt{2} \approx 1.41$) (Monofia 17) $\{1.59 , 4.41\}$

2 **Graph** the function f where $f(x) = x^2 - 2x + 3$ over the interval $[-1 , 3]$, then from the graph , **find the solution set of the equation** : $x^2 - 2x + 3 = 0$ (Qena 19) $\varnothing$

3 **Find in $\mathbb{R}$ the solution set of the following equations using the general formula**

1. $\frac{3}{x} + \frac{5}{x+1} = 2$ (rounding the result to two decimal digits) $\{0.44 , 3.44\}$

2. $\frac{x+1}{x+2} = \frac{2x+3}{3x+4}$ (rounding the result to two decimal digits) $\{$[illegible]$\}$

4 If : $x = 3$ is the equation of the symmetry axis of the curve of the function f where $f(x) = x^2 + a x + 8$, Then, **find the solution set of the equation** : $f(x) = 0$ $\{2 , 4\}$

5 If : $(2 , -3)$ is the point of the vertex of the curve of the function f where $f(x) = x^2 + a x + b$, Then, **find the solution set of the equation** : $f(x) = 0$ $\{-0.24 , 4.24\}$

6 If the solution set of the equation : $x^2 + a x + b = 0$ is $\{3 , -5\}$, then : **Find the values** of a and b. $(a = 2 , b = -15)$

7 If $x = 3$ is one of the two roots of the function : $x^2 + a x + b = 0$ and $a - b = 1$, then : **Find** the other root. (-1)

8 **Find the solution set of the equation** in $\mathbb{R}$: $(x + 2)^4 + 16 = 5x^2 + 20x$ $\{0 , -4 , -1 , -3\}$

SOLVING TWO EQUATION OF THE FIRST DEGREE IN TWO VARIABLES

1 Find in $\mathbb{R} \times \mathbb{R}$ the solution set of the two equations :

[1] $x - y = 0$ and $\frac{x}{y} = 4$ (Ismaillia 18 , Dakahlia 19) $\{(2,2),(-2,-2)\}$

[2] $x - y = 1$ and $x^2 + y^2 = 25$ (Port Said 18 , Aswan 19 , Giza 21) $\{(-3,-4),(4,3)\}$

[3] $x - y = 0$ and $x^2 + xy + y^2 = 27$ (Alex 19 , Kalyoubia 21) $\{(3,3),(-3,-3)\}$

[4] $y - x = 3$ and $x^2 + y^2 - xy = 13$ (Kalyoubia 17 , S-Sini 21) $\{(1,4),(-4,-1)\}$

[5] $y + 2x = 7$ and $(y + 2x - 8)^2 + x^2 = 5$ (Dakahlia 21) $\{(2,3),(-2,11)\}$

[6] $x + y = 2$ and $\frac{1}{x} + \frac{1}{y} = 2$ where $x \neq 0, y \neq 0$ (Menia 19) $\{(1,1)\}$

2 The difference between two numbers is 5 and the product of them is 36 .

Find the two numbers (Giza 12) (4 and 9)

3 The sum of two integers is 9 and the difference between their squares is 27 .

Find the two numbers (Dakahlia 12) (6 and 3)

4 A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm.

Find the lengths of the other two sides. (Monofia 15) (5 cm. , 12 cm.)

5 A right-angled triangle of hypotenuse length 13 cm. and its perimeter is 30 cm.

Find the lengths of the other two sides. (Monofia 15) (5 cm. , 12 cm.)

SOLVING TWO EQUATION OF THE FIRST DEGREE IN TWO VARIABLES

1 Find in $\mathbb{R}$ the set of zeroes of the following functions:

[1] $f(x) = x^2 - 2x + 1$ (S-Sinai 21) $\{1\}$

[2] $f(x) = x^3 + x^2 - 20x$ (Beni-Suef 21) $\{0,-5,4\}$

[3] $f(x) = (x-2)(x+3) + 4$ (Monofia 15) $\{-2,1\}$

[4] $f(x) = x^3 - 3x^2 - 4x + 12$ $\{-2,2,3\}$

2 If the set of zeroes of the function f where $f(x) = ax^2 + bx + 15$ is $\{3,5\}$

, then : **Find the values** of a and b. (Fayoum 19) ($a = 1$, $b = -8$)

3 If the set of zeroes of the function f where $f(x) = ax^2 + x + b$ is $\{0,1\}$

, then : **Find the values** of a and b. (Alex 17) ($a = -1$, $b = 0$)

2 ALGEBRIC FRACTIONS

The domain of the algebraic fractions – The common domain

Reducing the algebraic fractions – Operations on the algebraic fractions.

ALGEBRAIC FRACTIONAL FUNCTION

1 Find n (x) in the simplest form , showing its domain.

[1] $n(x) = \frac{x^2 - 4}{x^3 - 8}$

[2] $n(x) = \frac{x^3 + 1}{x^3 - x^2 + x}$

[3] $n(x) = \frac{x^3 + x^2 - 2}{x + 1}$

2 If : $n_1(x) = \frac{2x}{2x + 4}$, $n_2(x) = \frac{x^2 + 2x}{x^2 + 4x + 4}$.

Prove that : $n_1 = n_2$ (Menia 17 , Beheira 19 , Red sea 21)

3 If : $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^3 + x^2 + x}{x^4 - x}$.

Prove that : $n_1 = n_2$ (Kafr El Sheikh 17 , Souhag 19 , South Sini 21)

4 If : $n_1(x) = \frac{x^2}{x^3 - x^2}$, $n_2(x) = \frac{x^2 + x + 1}{x^3 - 1}$.

Prove that : $n_1 = n_2$ for the values of x belong to the common domain. (Cairo 19 , Assiut 21)

5 If : $n_1(x) = \frac{x^2 + x - 6}{x^2 - 4}$, $n_2(x) = \frac{x^2 - 9}{x^2 - x - 6}$.

Show whether : $n_1 = n_2$ or not (Dakahlia 17 , Ismailia 21)

OPERATIONS ON THE ALGEBRAIC FRACTIONS

1 Find n (x) in the simplest form , showing the domain of n where :

[1] $n(x) = \frac{x}{x - 4} - \frac{x + 4}{x^2 - 16}$. (Damietta 11 , Aswan 16 , North Sini 17 , Kalyoubia 18 , Red Sea 21 , Giza 21)

[2] $n(x) = \frac{x^2 + 3x}{x^2 + 2x - 3} - \frac{x - 2}{x^2 - 3x + 2}$. (Dakahlia 17 , Suez 18)

[3] $n(x) = \frac{x^2 - 2x + 4}{x^3 + 8} + \frac{x^2 - 1}{x^2 + x - 2}$. (Assiut 08 , Damietta 19)

[4] $n(x) = \frac{x}{x^2 + 2x} + \frac{x + 2}{x^2 - 4}$. (Aswan 12 , Sharkia 14 , Souhag 15 , Port Said 21)

[5] $n(x) = \frac{3x - 4}{x^2 - 5x + 6} + \frac{2x + 6}{x^2 + x - 6}$. (Cairo 11 , Cairo 12 , Beheira 14 , Qena 17)

6 $n(x) = \dfrac{x^2+2x+4}{x^3-8} - \dfrac{9-x^2}{x^2+x-6}$. (Beheira 15 , Alex 17 , Sharkia 17 , Monofia 18)

7 $n(x) = \dfrac{x^2+x}{x^2-1} - \dfrac{5-x}{x^2-6x+5}$. (Luxor 17 , Menia 18 , Dakahlia 19 , Souhag 21)

8 $n(x) = \dfrac{x^2+2x+1}{2x-8} \times \dfrac{x-4}{x+1}$. (Ismailia 15 , Cairo 16 , Suez 17)

9 $n(x) = \dfrac{x^3-1}{x^2-2x+1} \times \dfrac{2x-2}{x^2+x+1}$. (Souhag 12 , Luxor 17 , Fayoum 17 , Monofia 18 , Kalyoubia 18 , Dakahlia 19 , Beheira 21)

10 $n(x) = \dfrac{x-1}{x^2-1} \div \dfrac{x^2-5x}{x^2-4x-5}$. (Aswan 14 , Beheira 15 , Menia 16 , Matrouh 19)

11 $n(x) = \dfrac{x^2-2x-15}{x^2-9} \div \dfrac{2x-10}{x^2-6x+9}$. (Alex 16 , Beheira 18 , Gharbia 18)

12 $n(x) = \dfrac{x^2-2x+1}{x^3-1} \div \dfrac{x-1}{x^2+x+1}$. (Alex 11 , Qalyubia 12 , Gharbia 17 , Dakahlia 18 , Suez 19)

13 $n(x) = \dfrac{x^2-49}{x^3-8} \div \dfrac{x+7}{x-2}$ and find $n(1)$. (Gharbia 12 , Beheira 17 , Assiut 19 , Fayoum 19)

14 $n(x) = \dfrac{x^3-8}{x^2+x-6} \times \dfrac{x+3}{x^2+2x+4}$. (Souhag 19 , Red Sea 19 , Red Sea 21 , Dakahlia 21)

15 $n(x) = \dfrac{x-3}{x^2-7x+12} - \dfrac{x-3}{3-x}$. (Assiut 19 , Luxor 19 , Alex 21)

2 If the domain of the Function n where $n(x) = \dfrac{b}{x} + \dfrac{9}{x+a}$ is $\mathbb{R} - \{0, 4\}$, $n(5) = 2$.

Find : the value of a and b. (Menia 14 , Beheira 15 , Kafr El Sheikh 16 , South Sini 17 , Sharkia 19)

3 If : $n(x) = \dfrac{x^2-2x}{(x-2)(x^2+2)}$.

1 **Find :** $n^{-1}(x)$, showing its domain.

2 If : $n^{-1}(x) = 3$, **What :** the value of x (Aswan 16 , Gharbia 17 , Port Said 17 , Kalyoubia 18 , Alex 19)

4 If the set of zeroes of the function f where $f(x) = \dfrac{ax^2-6x+8}{bx-4}$ is $\{4\}$ and its domain is $\mathbb{R} - \{2\}$

Find : the value of a and b. (Sharkia 17)

5 If the domain of n : $n(x) = \dfrac{L}{x} + \dfrac{9}{x-m}$ is $\mathbb{R} - \{0, -2\}$, $n(4) = 1$.

Find : the value of L and m. (Menia 17)

3 PROBABILITY

Operation on the events

(union – intersection – difference and complement)

1 If : A and B are two mutually exclusive events in a sample space of a random experiment and $P(A) = \frac{1}{2}$, $P(B) = \frac{1}{3}$, Then **Find** $P(A \cup B)$. (Aswan 17 , Qena 18 , Gharbia 18) ($\frac{5}{6}$)

2 If : A and B are two events in a sample space of a random experiment , $P(B) = \frac{1}{12}$, $P(A \cup B) = \frac{1}{3}$, Then **Find** $P(A)$ if :

[1] A and B are two mutually exclusive events

[2] $A \subset B$ (Cairo 11 , North Sini 14 , Luxor 17 , Kafr El Sheikh 17 , Port Said 18) « $\frac{1}{4}$, $\frac{1}{3}$ »

3 A box contains 12 balls , 5 of them are blue , 4 are red and the left are white. A ball is randomly drawn from the box . **Find** the probability that the drawn ball is :

[1] Blue [2] Not red [3] Blue or red (North Sini 12 , Alex 13 , Luxor 18 , Souhag 18) « $\frac{5}{12}$, $\frac{2}{3}$, $\frac{3}{4}$ »

4 If : X and Y are two events in a sample space of a random experiment where :

$P(Y) = \frac{2}{5}$, $P(X) = P(X')$, $P(X \cap Y) = \frac{1}{5}$ Then **Find** :

[1] $P(X)$ [2] $P(X \cup Y)$ (Dakahlia 14 , Kalyoubia 16 , Kafr El Sheikh 18) « $\frac{1}{2}$, $\frac{7}{10}$ »

5 If : A and B are two events in a sample space of a random experiment , $P(A) = 0.7$, $P(B) = 0.4$, $P(A \cap B) = 0.2$, Then **Find** :

[1] $P(A')$ [2] $P(A \cup B)$ (Cairo 12) « 0.2 , 0.9 »

6 If : A and B are two events in a sample space of a random experiment , $P(A) = 0.6$, $P(B) = 0.3$, $P(A \cap B) = 0.2$, Then **Find** :

[1] $P(A \cup B)$ [2] $P(A - B)$ (Giza 12) « 0.7 , 0.4 »

7 A bag contains 20 identical card numbered from 1 to 20. A card is randomly drawn. **Find** the probability that the number on the card is :

[1] Divisible by 3 [2] An odd and divisible by 5 (Sharkia 12) « $\frac{3}{10}$, $\frac{1}{10}$ »

8 If : A and B are two events in a sample space of a random experiment. , $P(A) = 0.8$, $P(B) = 0.7$, $P(A \cap B) = 0.6$, Then **Find** :

[1] The probability of non–occurrence of the event A

[2] The probability of occurrence of the two events at least

[3] The probability of occurrence of one event without the other

(Gharbia 17 , Sharkia 17 , Kalyoubia 19 , Beheira 19 , Beheira 21) « 0.2 , 0.9 , 0.3 »

Best wishes, mr Abdelrahman Essam